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How the air slows a closing book

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How the air slows a closing book

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When two flat surfaces approach each other, the fluid in between is accelerated and ejected from the sides at large speeds. This situation occurs often in everyday life, such as when you step in a puddle and create splashes of water, or when you clap your hands or close a book and create jets of air. For these systems, the inertia of the fluid resists the acceleration, creating large nonlinear forces on the flat surfaces. In this work, we study the case of a closing book. The fluid motion in this case is relatively easy to model, using the conservation of mechanical energy, and to measure using a MEMS gyroscopic sensor. This study reveals the unusual forces that occur when two plates collide in an experiment that can be performed by students at home. © 2024 Published under an exclusive license by American Association of Physics Teachers.

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I. INTRODUCTION

When you open a book by lifting its cover, you may find that a rather large initial force is needed to “unstuck” the cover from the pages underneath it. This “suction” force is due to the pressure of the surrounding air on the cover that opposes the upward motion. This is one example of a general effect: the magnification of the fluid force on an object moving towards or away from a rigid boundary.^{1–4} This effect occurs because rigid boundaries restrict the fluid flow, thus increasing the fluid velocities and, therefore, the pressure differences. This effect can be easily observed in many everyday situations such as a beach ball bouncing on the floor.⁵ It is also important in aeronautics for the take-off and landing of airships⁶ or for the motion of a ship in shallow water⁷ or near the shore.⁸ Here, we consider this effect for the case of a closing book, where the two flat boundary surfaces result in especially large forces.

The closing book example is useful for teaching for many reasons. First, it turns out to be easy to model. The torque exerted by the fluid on the rotating book cover can be calculated relatively easily for small opening angles, and an analytical solution for the falling cover derived. This system also provides an example of a variable mass system, since the mass of the fluid in the gap between the book and its cover is changing as the cover closes. Such systems are popular in education because they provide a deeper understanding of Newton’s laws. Finally, the motion of the rotating cover can be easily measured using the gyroscopic sensor in a smartphone.^{9,10} Thus this experiment is ideal for students to do at home.

This paper is organized as follows: Section II describes the theoretical model:¹¹ Sec. II A describes the model for free-fall motion, Sec. II B justifies the approximation that the air is an ideal fluid and derives the torque acting on the rotating plate by the air, and Sec. II C discusses the motion of the falling plate and derives the analytical solution when the rotational inertia of the plate is negligible compared to the rotational added mass of the air. Section III details the experiment, presents the experimental results, compares them to the theoretical predictions, and discusses how students may do the experiment at home with a smartphone and book. Section IV gives a brief summary and details possible extensions of the present results. A sample student lab and more details can be found in the supplementary material.¹²

II. THEORETICAL MODEL

A. Free-fall case

Let us first examine what the motion of a closing book would be if we neglect the forces exerted by the air on the cover.

The closing book is modeled as a fixed bottom surface and a mobile upper plate that can pivot about the hinge that connects them. The upper plate is inclined at an angle θ from the horizontal, see Fig. 1. The equation of motion for the falling upper plate is given by Newton’s second law for rotational motion. Neglecting the effects of the air, this is

$$I_P \frac{d^2\theta}{dt^2} = -\Gamma_{\text{gravity}}. \quad (1)$$

Here I_P is the rotational inertia of the plate and Γ_{gravity} is the torque on the plate from gravity, which is independent of θ in the limit $\theta \ll 1$. Thus, the angular acceleration is constant, and the motion is equivalent to free-fall. Assuming the plate is released from rest at an initial angle θ_0 , the angular position and velocity are

$$\theta = \theta_0 - \frac{1}{2} \frac{\Gamma_{\text{gravity}}}{I_P} t^2 \quad (2)$$

and

$$\frac{d\theta}{dt} = -\frac{\Gamma_{\text{gravity}}}{I_P} t. \quad (3)$$

The time for the plate to fall to horizontal is

$$t_{\text{fall}} = \sqrt{\frac{2\theta_0 I_P}{\Gamma_{\text{gravity}}}}, \quad (4)$$

and the angular speed at that time, which is the maximum speed reached by the plate, is

$$\left| \frac{d\theta}{dt} \right|_{\text{max}} = \sqrt{2 \frac{\Gamma_{\text{gravity}}}{I_P} \theta_0}. \quad (5)$$

B. Deriving the torque due to the air

Let us now examine how the air’s motion around the falling plate affects its fall.

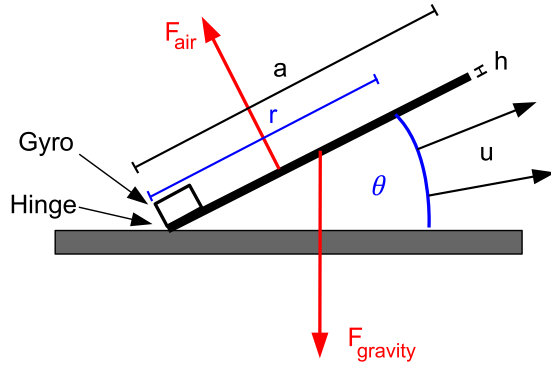


Fig. 1. Sketch of the experiment showing the forces acting on the upper plate that create a torque about the hinge. Gravity causes the upper plate to rotate downward, expelling the air in the gap with radial speed $u(r,t)$. This motion is opposed by the force of the air on the upper plate. The length radial from the hinge is r , which has a maximum value of a , h is the thickness of the plate, and θ is the angle between the plate and the horizontal surface. The gyroscopic sensor is placed close to the hinge.

Initially, the air and the plate are assumed to be at rest. When the upper plate is released, it rotates clockwise, decreasing the angle θ between the surfaces, thereby ejecting the air out from the gap between the plate and the horizontal surface.

Let us examine the influence of the air's viscosity on the air flow. As the air moves, friction between the surfaces and the air close to the surfaces will decrease the air's velocity components parallel to the surfaces. Internal friction in the fluid causes this slowing of the air to spread from the surfaces out into the air gap. The timescale for momentum to diffuse a distance d is given by $t_{\text{diffusion}} = d^2/\nu$, where ν is the kinematic viscosity constant.³ Taking $d = r\theta_0$ = the initial width of the gap at a distance r from the hinge, and using $\nu = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$, this gives

$$t_{\text{diffusion}} = 6.5 \times 10^4 \frac{\text{s}}{\text{m}^2} (r\theta_0)^2. \quad (6)$$

Evaluating this at the opening of the gap where r is its maximum value $= a = 0.277 \text{ m}$ for this experiment, and taking $\theta_0 = 0.1 \text{ rad}$, then $t_{\text{diffusion}} = 50 \text{ s}$. This time is much larger than the free-fall time $t_{\text{fall}} \approx \sqrt{4\theta_0 a/3g} \approx 0.06 \text{ s}$, where Eq. (4) was used and a uniform plate was assumed. Thus, the effects of the air's viscosity on the flow inside the gap are negligible. This has two implications for the following. First, the energy dissipated by the internal friction can be neglected. Second, the slowing of the air near the surfaces can be ignored. (This is known as slip boundary conditions.)

Now let us find the air's velocity. To calculate this, we shall assume that the density of the air is a constant. This is a reasonable assumption if the air's velocity is much slower than the speed of sound in air, and this will be shown to be the case for this experiment by calculations made using this assumption. However, we also know this assumption to be reasonable by the observation that when a book falls closed through a small angle, it typically does not produce the loud "crack" sound associated with a shock wave.

The assumption of constant density means that air volumes are conserved. Then the air flow rate out of the gap is given by the rate of change in the wedge-shaped volume of the air gap as the plate rotates. Consider the volume inside a radius r , $V(r,t) = r^2 L \theta(t)/2$ where L is the length of the plate

in the direction parallel to the rotation axis. Neglecting the flow of air from the short ends of the plate, then the volume flow rate is $-uLr\theta$, and one obtains

$$u(r,t) = -\frac{1}{2} r \frac{1}{\theta} \frac{d\theta}{dt}, \quad (7)$$

with $u(r,t)$ being the average radial fluid velocity. As expected, the speed of the air in the gap becomes large as θ decreases. Equation (7) treats the air flow as approximately two-dimensional, which is reasonable if $L \gg a$. For small angles θ , the air speed component perpendicular to the plate (of order the plate speed $rd\theta/dt$) is much smaller than the average radial speed and can, therefore, be neglected. Since slip boundary conditions are appropriate here, it is reasonable to take the radial velocity as being uniform across the gap.

Another equation describing the air's motion is given by the change in mechanical energy. Let us denote the torque exerted by the air on the plate as Γ_{air} . As the plate rotates, and by virtue of Newton's third law, the change in mechanical energy can be written as

$$-\Gamma_{\text{air}} \frac{d\theta}{dt} = \frac{dK_{\text{inside}}}{dt} + S_{\text{out}}, \quad (8)$$

where K_{inside} is the kinetic energy of the air contained within the gap and S_{out} is the rate of energy flowing out of the gap. Before deriving the various terms, let us make a few comments.

Equation (8) neglects the kinetic energy of the air above the plate because the air speeds there are much smaller than those between the plates. The S_{out} term in Eq. (8) turns the system from an energy conserving system, where the falling plate would bounce off the surface and move back up to its original height, into an irreversible system where the energy leaving the gap is lost to the system, and the plate settles smoothly to the horizontal surface. This agrees with the common experience of closing a book, and with the patterns of flow created in fluids with small viscosities ejected from small orifices. Then the flow does not resemble that of an ideal fluid, spreading outward uniformly in all directions, but instead is turbulent and produces structures such as a vortex ring and a jet^{13,14} that irreversibly transport energy away from the gap.

The kinetic energy of the air inside the gap is evaluated by using the kinetic energy per unit volume, $\rho u^2/2$, where ρ is the density of the air. Using Eq. (7), and integrating the kinetic energy density over the volume of air in the wedge gives

$$K_{\text{inside}} = \frac{1}{32} \rho L a^4 \frac{1}{\theta} \left(\frac{d\theta}{dt} \right)^2. \quad (9)$$

The rate of energy flow out of the end of the air gap is given by the kinetic energy per unit volume, $\rho u^2/2$, multiplied by the volume flow rate, $uLr\theta$, evaluated at the edge of the gap at $r = a$. This gives

$$S_{\text{out}} = -\frac{1}{16} \rho L a^4 \frac{1}{\theta^2} \left(\frac{d\theta}{dt} \right)^3. \quad (10)$$

The minus sign in Eq. (10) makes S_{out} a positive quantity when the plate is falling. Using Eqs. (7)–(10), the expression for the torque is

$$\Gamma_{\text{air}} = \frac{1}{16} \rho L a^4 \left[\frac{3}{2} \left(\frac{1}{\theta} \frac{d\theta}{dt} \right)^2 - \frac{1}{\theta} \frac{d^2\theta}{dt^2} \right]. \quad (11)$$

The second term on the right-hand side of this equation increases the effective inertia of the rotating plate, slowing down its motion. This term is usually called the added mass (actually, the rotational inertia¹⁵) term. The added mass corresponds to the increased effective inertia of an accelerating object due to the fact that the object also accelerates the fluid around it. The added mass conserves energy. The added mass force is not usually discussed in introductory physics courses, but it should be because it is relevant for many everyday situations, is easy to understand theoretically, and is easy to measure in the introductory physics lab.^{16–18}

The first term on the right-hand side of Eq. (11) occurs because the added mass is varying⁵ and because energy leaks out from the gap. It is always positive, no matter in which direction the plate is moving, and as a consequence it tends to make the upper plate move away from the lower one. Thus both terms in Eq. (11) act to slow the motion of the falling plate.

The above derivation used a mechanical energy framework, but Eq. (11) can also be derived using other methods. For example, Ref. 1 found the forces on colliding plates by using the equivalent of Eq. (7) and solving Euler's fluid equation to find the pressure distribution inside the gap. Yih⁴ studied many different colliding plate geometries by solving Laplace's equation in the gap for the velocity potential and then using the unsteady Bernoulli equation to find the pressure there. These methods assume the pressure everywhere outside the gap is atmospheric pressure.

C. Solving for the motion

The equation of motion for the falling upper plate is given by Newton's second law for rotational motion,

$$I_P \frac{d^2\theta}{dt^2} = \Gamma_{\text{air}} - \Gamma_{\text{gravity}}, \quad (12)$$

which is Eq. (1) with the addition of the torque due to the air. The buoyant force is neglected here because the objects used in this experiment have much larger densities than the air. Substituting into Eq. (12) the expression for Γ_{air} given in Eq. (11), and rewriting the equation in terms of scaled variables gives

$$\left[\delta + \frac{1}{x} \right] \frac{d^2x}{d\tau^2} = \frac{3}{2} \left(\frac{1}{x} \frac{dx}{d\tau} \right)^2 - 1. \quad (13)$$

Here, the dimensionless angle is $x = \theta/\theta_0$, and the dimensionless time is $\tau = t\sqrt{16\Gamma_{\text{gravity}}/(\rho L a^4)}$. The quantity

$$\delta = \frac{16I_P\theta_0}{\rho L a^4} \quad (14)$$

is the single, dimensionless parameter on which the dynamics depend. Physically, δ is the ratio of the rotational inertia of the plate to the initial rotational added mass due to the air.

It is interesting to consider how the motion depends on the different parameters. In particular, δ is independent of gravity, so while the value for the acceleration of gravity is relevant for determining the timescale (since it is in Γ_{gravity}) it is

not relevant for the dynamics. This behavior is similar to that of free-fall motion. However, in contrast to free-fall motion, the mass of the plate does not cancel out but instead enters both in the timescale and δ . This is because there are two mass scales: those of the plate and of the air in the gap. For a uniform plate, δ can be rewritten as $\delta_{\text{uniform}} = (16/3)(\rho_p/\rho)(h/a)\theta_0$ and the timescale as $t/\tau_{\text{uniform}} = \sqrt{(1/8)(\rho/\rho_p)(a^2/hg)}$, where h is the thickness of the plate and ρ_p is its density. These expressions are independent of L but do depend on a , the width of the plate.

For arbitrary δ , Eq. (13) must be solved numerically to find the position or velocity of the plate as a function of time over the complete range of motion. However the general form of these numerical solutions is easy to understand, and analytical solutions exist for various limits.

When the plate is released from rest, at $x(0) = 1$, the first term on the right-hand side of Eq. (13) vanishes, so the magnitude of the initial angular acceleration is approximately $1/(1+\delta)$. Thus the speed initially increases linearly in time with this slope. As the speed increases, and x decreases, the neglected term grows and eventually causes the acceleration to vanish. The maximum speed is then reached at $x = x_{\text{max}}$,

$$\left| \frac{dx}{d\tau} \right|_{\text{max}} = \sqrt{\frac{2}{3}} x_{\text{max}}. \quad (15)$$

After this point in time the decreasing x keeps the right-hand side of Eq. (13) positive, and so the plate smoothly decelerates to $x = 0$. Thus the angular speed as a function of time is predicted to be a one-humped function.

An analytical solution for the complete motion exists when $\delta = 0$. This corresponds to the torque due to gravity being balanced by the torque due to the air. δ is negligible when the initial angle is very small, or when the rotational inertia of the plate is small (see Eq. (14)), or at late times when x is small (see Eq. (13)).

When $\delta = 0$, Eq. (13) can be rewritten as

$$\frac{d^2(x^{-1/2})}{d\tau^2} = \frac{1}{2} x^{-1/2}. \quad (16)$$

Using that the plate is released from rest at $x(0) = 1$, the solution for the position is

$$x(\tau) = \frac{1}{\cosh^2[\tau/\sqrt{2}]}. \quad (17)$$

As time goes to infinity, Eq. (17) predicts that the plate exponentially relaxes to $x = 0$. The angular velocity as a function of time is easy to derive from Eq. (17). The maximum dimensionless speed for the $\delta = 0$ analytical solution is

$$\left| \frac{dx}{d\tau} \right|_{\text{max}} = \left[\frac{2}{3} \right]^{3/2}. \quad (18)$$

Equation (18) predicts that the maximum angular speed, $|d\theta/dt|_{\text{max}}$, is proportional to the initial angle, θ_0 , since x is scaled by θ_0 .

For comparison, the free-fall in vacuum equations for the rotating plate, Eqs. (2), (3), and (5), in terms of the scaled variables, are $x(\tau) = 1 - \tau^2/(2\delta)$, $dx/d\tau = -\tau/\delta$, and $|dx/d\tau|_{\text{max}} = \sqrt{2/\delta}$.

Although it is not measured in the experiment we have performed, it is interesting to consider the prediction of the model for the speed of the air at the edge of the gap ($r = a$) and for very late times. Then $x \ll \delta$, and both the gap size and the plate speed decay exponentially, see Eq. (17). Then Eq. (7) predicts a steady-state air flow speed of $|u(a, \infty)| = \sqrt{8\Gamma_{\text{gravity}}/(\rho L a^2)}$, independent of the initial angle. Assuming a uniform plate, then this becomes

$$|u(a, \infty)|_{\text{uniform}} = \sqrt{\frac{4\rho_p h g}{\rho}}. \quad (19)$$

It is interesting that this speed does not depend on the width of the plate, a , but only on the plate's average density per unit area. For $\delta = 0$, this is the peak flow speed, but for $\delta > 0$, the peak flow speed occurs before $x = 0$. Evaluating Eq. (19) for this experiment¹⁹ gives $|u(a, \infty)| = 5.0$ m/s, much less than the speed of sound, as assumed earlier.

The analysis presented here breaks down at very small gap sizes because of a variety of effects: irregularities on the surfaces, viscous effects (see Eq. (6)), and the breakdown of the assumption that the air is a continuous fluid.²⁰

III. EXPERIMENTAL METHODS AND RESULTS

While a smartphone and book were used for preliminary measurements, the data reported here used an idealized arrangement, see Fig. 2. A low-density, rectangular plate of corrugated cardboard, with a large length to radial width ratio, was taped at one edge to a table. By making the length to width ratio of the plate relatively large, the flow of air was approximately two-dimensional and so better matches the assumptions of the theoretical model. Also, by using a low-density piece of cardboard and subsequently attaching mass to the cardboard, a wide range of parameters can be studied. The experiment consists of raising the end of the cardboard opposite the hinge a few centimeters, releasing it from rest, and measuring the angular velocity as a function of time as it falls.

Corrugated cardboard was chosen because it is relatively flat, has low density, and yet is rigid enough that there was no visible sagging when supported in the middle by an instrument (a screwdriver in Fig. 2) before release. The length L of the cardboard was determined by the largest length readily available, and a was then chosen to be small enough that the flow was approximately two dimensional, but not so small that small δ values would be difficult to

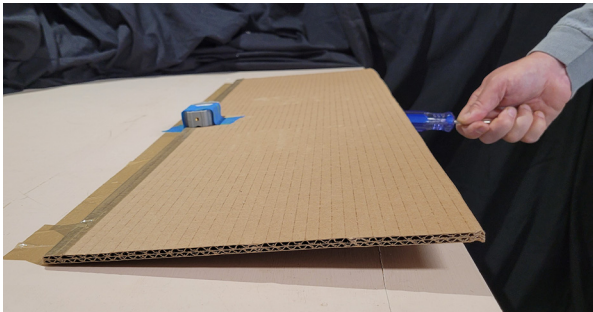


Fig. 2. Photograph of the experiment. A rectangular piece of cardboard is taped along the long left edge to a table, a wireless gyrometer is taped to the cardboard near the hinge, and a screwdriver handle is about to be quickly removed from the open, right edge.

Table I. Physical parameters of the experiment. L is the distance parallel to the hinge, a is the distance perpendicular to the hinge, and h is the thickness.

Dimensions	Cardboard	Sensor	Phone
L (cm)	87.3	7.5	16.4
a (cm)	27.7	4.0	8.25
h (cm)	0.62	2.0	1.0
Mass (g)	175.9	34.64	229.0

achieve. The tape used for the hinge was thin, flexible packaging tape. It functions here as a hinge and also to prevent the flow of air from this edge of the cardboard.

The rotational motion of the falling cardboard was measured using a PASCO wireless acceleration/altimeter, PS-3223, that was taped to the cardboard with low residue painter's tape, with the long edge next to the hinge, see Fig. 2. This device contains a micromechanical systems (MEMS) gyroscopic sensor, similar to those found in smartphones, that measures angular velocity as a function of time. A sampling rate of 1000 Hz was used. The sensor's sensitivity to small angular velocity measurements was limited by fluctuations with a standard deviation of 0.040 rad/s when the sensor was at rest. All measurements were transmitted wirelessly to a laptop using PASCO's Capstone software, and this software was then used to determine the initial opening angle, the maximum velocity, and the angle at the maximum velocity.

The mass and mass distribution of the plate was varied by taping a smartphone to seven different positions on the cardboard. This changed the rotational inertia and the torque due to gravity. The phone was also sometimes used as a motion sensor to qualitatively confirm the main features of the plate's motion. Table I gives the masses and dimensions of the objects used in the experiment. Table II gives the positions of the smartphone, the corresponding moment of inertia, I_P , as calculated from the parameters in Table I, and the torque due to gravity, Γ_{gravity} , for each mass distribution. These calculations treated the board, phone, and sensor as uniform objects.

The plate was smoothly released from rest by first propping it open with a small, rigid, smooth object inserted in the middle of the open end of the plate, and then quickly removing this object, see Fig. 2. Thirteen different objects were used (ranging from the blunt end of a pen to a large light-bulb) to produce opening angles spanning the range 0.02–0.34 radians, for each of the seven mass distribution,

Table II. Dynamical parameters for different mass distributions.

Distance from the hinge to the smartphone's long, inner edge (cm)	Moment of inertia I_P (10^4 g cm ²)	Torque due to gravity, Γ_{gravity} (10^6 g cm ² /s ²)	δ/θ_0	τ/t (1/s)
No phone	4.51	2.46	12.3	25.9
5.25	6.65	4.56	18.1	35.3
7.70	7.84	5.12	21.4	37.3
10.75	9.70	5.80	26.5	39.8
13.10	11.4	6.33	31.2	41.5
16.55	14.4	7.11	39.3	44.0
19.05	16.9	7.67	46.2	45.7

resulting in 91 datasets. Only small initial angles were used so that the approximations made in the theoretical model were valid over the entire range of motion.

The density of the air, $\rho = 1.150 \text{ kg/m}^3$, was determined by measuring the air pressure, temperature and relative humidity, and then using an online air density calculator.²¹ The air density, with the mass and position parameters, was used to calculate the dimensionless scaling parameters, δ and τ . The scaling factors for these parameters are also given in Table II.

Figure 3 shows the measured dimensionless angular velocity ($-dx/d\tau$) vs time (τ), and the predictions of the model, for two different plate closings. The points are the data, and the curves are the numerical solutions of Eq. (13). The model predicts the angular velocity to be a one-humped function, initially increasing linearly in time with a slope of $1/(1+\delta)$, with an exponential fall off at late times that is independent of δ . The theoretical curve in Fig. 3 for the smaller δ is close to that of the $\delta=0$ model, obtained by differentiating Eq. (17). Agreement between the data and the theoretical prediction for the small θ_0 value (open circles and solid curve) is very good, considering that there are no free parameters and that the model assumes $L/a = \infty$ while it is actually 3.15. For the larger θ_0 value (closed circles and dashed curve), the peak occurs at a later time than predicted. This deviation may be due to neglecting the kinetic energy of the air above the gap, which underestimates the initial added mass, and this approximation is worse at large angles. Most of the fluctuations in the data in Fig. 3 are due to the inherent noise of the MEMS sensor. However the data for the larger θ_0 value (closed circles) has a significant, negative fluctuation at $\tau \approx 6.3$. This is a small rebound of the plate from the table at late times. This rebound may be caused by the cardboard deviating from being perfectly flat and rigid.

Figure 4 shows a plot of the maximum angular speed ($|d\theta/dt|_{\max}$) vs the initial release angle (θ_0). These measurements are all for one mass distribution, $\delta/\theta_0 = 12.3$, corresponding to the situation when there was no smartphone on the plate. The measured maximum speeds agree with the solid curve, which is the prediction of the model, obtained by numerically solving Eq. (13). When θ_0 is small, the measured values are also close to the dotted line in Fig. 4, which is the analytical solution for $\delta=0$ given in Eq. (18). The

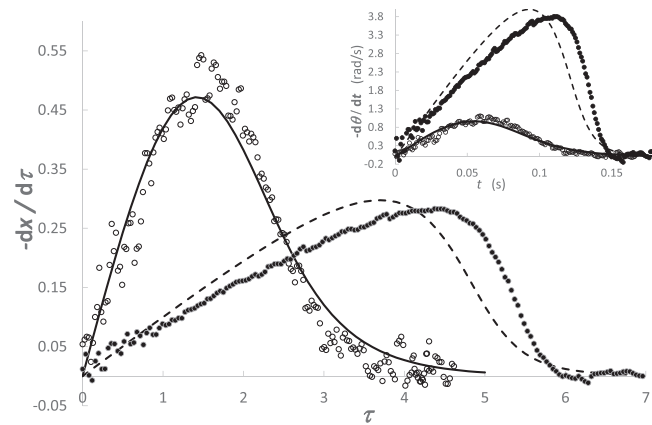


Fig. 3. Plot of angular velocity vs time. The main plot represents dimensionless quantities while the inset plots unscaled parameters. Two different plate closings are shown: $\delta=0.97$ and $\theta_0=0.079$ radians (open circles) and $\delta=8.9$ and $\theta_0=0.34$ radians (closed circles). The solid and dashed curves are the respective theoretical predictions.

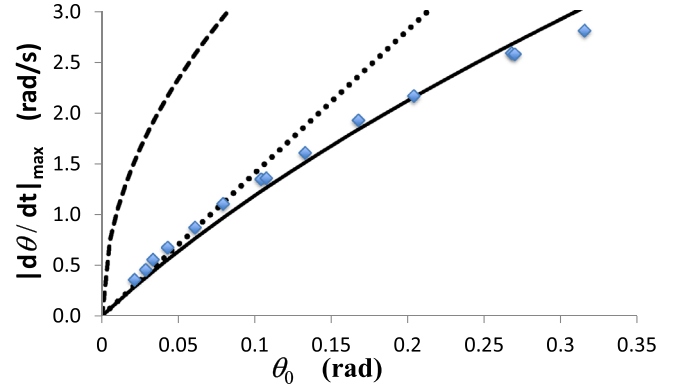


Fig. 4. Plot of the maximum angular speed vs the initial angle for one mass distribution with $\delta/\theta_0 = 12.3$. The solid diamonds are the measurements, and the curves are various theoretical predictions. The solid curve is a numerical solution of Eq. (13), the dotted line is the $\delta=0$ analytical solution, and the dashed line is for free-fall in vacuum.

measured values are far from the dashed curve, which is the prediction for free-fall in vacuum given in Eq. (5). There are no free parameters in these models.

Figure 5 plots the dimensionless maximum angular speed ($|dx/d\tau|_{\max}$) (top) and the dimensionless opening angle at this point ($x_{\max}$) (bottom) vs the dimensionless parameter δ given in Eq. (14). The markers are the data and the curves are the theoretical predictions, with the solid curves the model

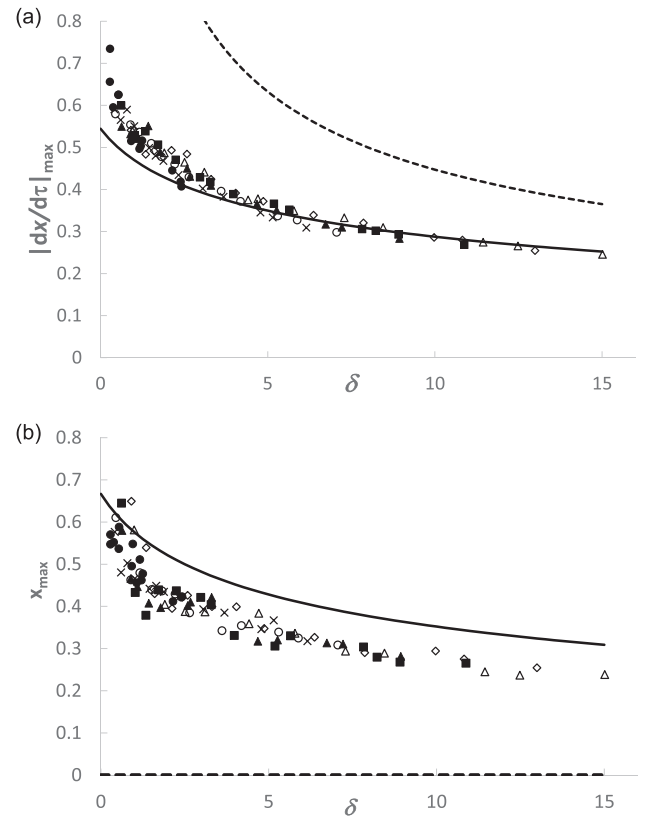


Fig. 5. Plot of the dimensionless maximum angular speed (top) and the dimensionless angle at which the maximum speed occurs (bottom) vs the dimensionless parameter δ . Identical markers correspond to experimental measurements with the same mass distribution but different initial angles: $\delta/\theta_0 = 12.3$ (solid circle), 18.1 (crosses), 21.4 (open circle), 26.5 (solid triangle), 31.2 (solid square), 39.3 (open diamond), and 46.2 (open triangle). The solid curves are the theoretical predictions using Eq. (13), and the dashed curves are the prediction without fluid forces, i.e., free-fall in vacuum.

developed here and the dashed curves the free-fall predictions. The solid curves are obtained by numerically solving Eq. (13); however the $\delta = 0$ values are also given by the analytical solution, see Eqs. (15) and (18). The different data symbols in Fig. 5 correspond to different mass distributions on the upper plate. For a fixed mass distribution, δ varies due to different initial opening angles. The relative uncertainty on $x_{\max}$ is larger than on $|dx/d\tau|_{\max}$ because of the broad-peaked shape of angular velocity vs time data, see Fig. 3. Figure 5 top is analogous to Fig. 4, but with dimensionless parameters and with multiple mass distributions.

The collapse of the datasets in Fig. 5 to a single curve shows that δ is a good parameter for describing the behavior. The data in Fig. 5 generally follow the theoretical predictions of the model developed here (solid curves), but there is some deviation towards the free-fall in vacuum predictions, especially for $x_{\max}$. Some of this deviation is probably due to the loss of air at the short ends of the plate. At large δ the deviations in $x_{\max}$ may also be related to the smaller initial acceleration seen in Fig. 3, which is probably due to neglecting the kinetic energy of the air above the upper plate.

The current experiment can easily be done by students, at home, with a book (or a cardboard) and a smartphone.¹² The smartphone app phyphox²² does an excellent job accessing the smartphone's MEMS sensor. This app will record the data, and can wirelessly transmit it to a computer where all of the subsequent analysis can be done on a spreadsheet. The results found using this method were similar to those presented here but with a few differences. It is harder to reach small values of δ because book covers are denser than corrugated cardboard, and because smartphones are more massive and have smaller sampling rates than the PASCO sensor. Also, the sampling rate of smartphones varies considerably.²³ Another difficulty for use of the experiment as an at-home lab activity is that direct comparison with the theoretical model requires knowledge of the mass of the book cover, and this can be hard to obtain at home. However, without knowing the mass of the cover, one can still observe the cushioning of the air on the falling cover (as shown in Fig. 3) and make plots such as that in Fig. 4 from which the mass of the cover can be determined. Also, a book typically has a smaller length to width ratio, and so the flow will be less two-dimensional.

IV. SUMMARY AND EXTENSIONS

We have shown that when two flat surfaces in air collide, the effect of the air is relatively easy to both measure and model. Measuring the angular velocity of a closing book with a MEMS sensor, such as found in most smartphones, finds that the angular speed increases linearly at early times (at a rate slower than predicted by free-fall) and decays exponentially at late times. The maximum plate speed is found to increase with the initial angle, but is much less than predicted for free-fall. This behavior is well described using a simple model of the air flow. Because this is a short duration flow, viscosity is negligible and the air is well approximated by an ideal fluid. For small angles and long plates, the speed is approximately uniform across the gap and the torque on the plate is easily calculable analytically. Both the model and the experiment show that the falling plate's motion depends on a single, dimensionless parameter δ , the ratio of the rotational inertia of the plate to the initial rotational added mass due to the air. The relevant fluid force here is purely due to the inertia of the air, and is commonly called the added mass force.

There are many possibilities for student research projects that build on or extend the work presented here. For example, one could study other geometries, such as a falling coffee filter colliding with the floor. When the coffee filter is close to the floor, the velocity of the air flow under it becomes large, analogous to the closing book. The cylindrical geometry of the coffee filter is identical to the theoretical models in Refs. 1, 3, and 4. Another extension of the current work would be to study the fluid flow outside the gap numerically for an ideal fluid. This might explain some of the small deviations found between the current model and the experimental results, and also would provide numerical visualizations of the airflow, which are useful for education. It would be interesting to compare these numerical visualizations with experimental observations of the airflow, made using flow tracers such as smoke or bubbles. Another experiment would be to study the closing book system for different values of L/a . This probes deviations from two-dimensional flow, which might also be responsible for the minor discrepancies between the model and the experiments. Another possible experiment would be to change the fluid by performing the experiment underwater with an aluminum plate or to study the suction case by turning the current experiment upside down and letting the plate fall open.²⁴ This latter case is similar to the physics of suction cups²⁰ and the lifting of objects from the sea floor where it is called the breakout force. The theoretical analysis in Sec. II should apply for this case,¹ but the inflow may be more like that of an ideal fluid. Asymmetries between inflow and outflow from an orifice make possible the jet propulsion of some aquatic animals, thus the current experiment and variations thereof (such as measuring the horizontal force on the system), could be used to illustrate the principles relevant for the scallop theorem.²⁵ As can be seen, from this simple experiment, one can imagine a number of similar labs aimed at introducing students to the everyday application of fluid mechanics.

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AUTHOR DECLARATIONS

Conflict of Interest

The author has no conflicts to disclose.

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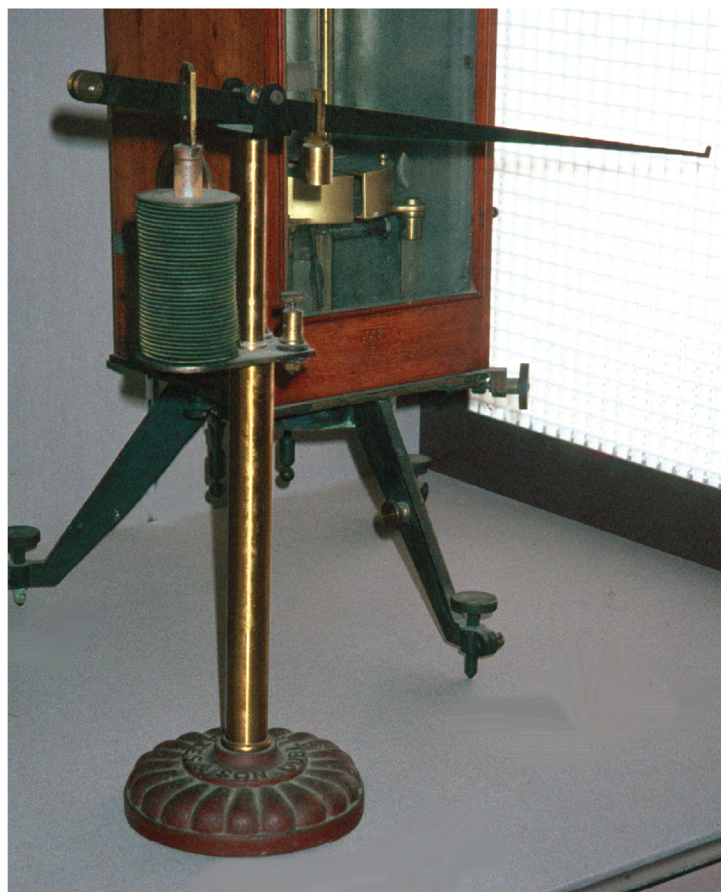
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- ¹²The supplementary material online contains (1) a student lab assignment based on the manuscript, (2) sample results for this lab made using a smartphone and a book cover, and (3) the Mathematica code used in the analysis.
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Magnetic Current Balance

This apparatus enables one to do an experiment in which the magnetic force on a slug of iron immersed in a current-carrying coil can be measured as a function of the current. The coil, of known mass, and the balancing weight are adjusted to keep the arm level. The cast-iron base of the device is typical of late 19th century design. Behind the instrument is a quadrant electrometer, used to measure electrostatic potentials. The instruments are at the Callan Museum at St. Patrick's College in Maynooth, Ireland. (Picture and text by Thomas B. Greenslade, Jr., Kenyon College)