



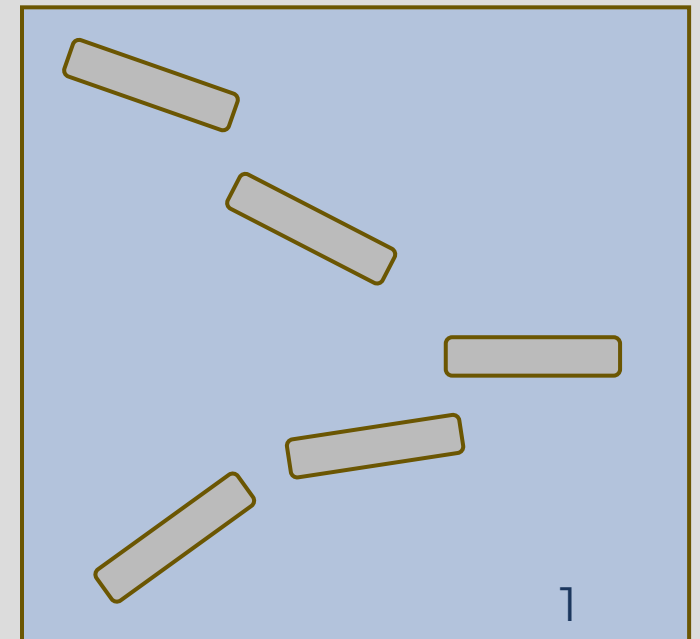
12. Autumn coin

Kryštof Basista, Czech Republic



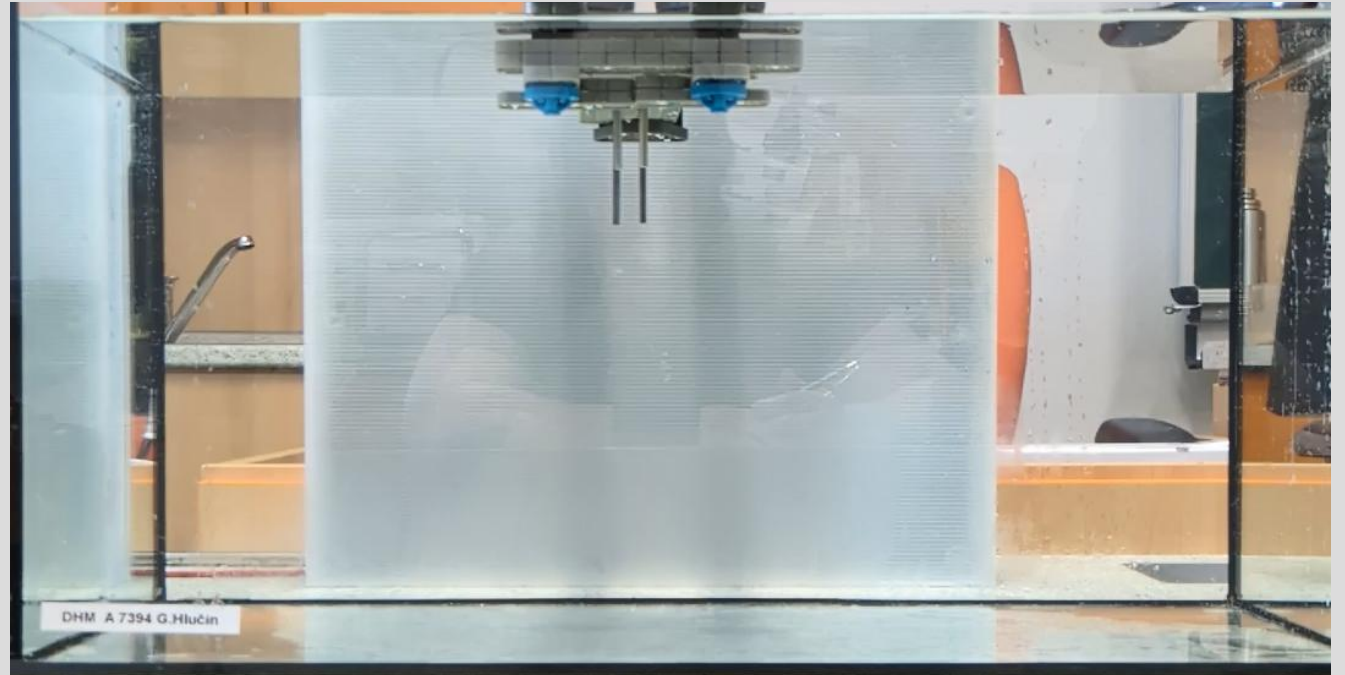
The motion of a coin falling to the bottom of a tank filled with liquid can be remarkably similar to the fluttering and tumbling of a falling autumn leaf. Investigate how the motion of the coin depends on relevant parameters.

(AI was strictly used for assistance in coding)



Problem statement

- The **motion of a coin** falling to the bottom of a tank filled with liquid can be remarkably similar to **the fluttering and tumbling of a falling autumn leaf**. Investigate how the motion of the coin depends on **relevant parameters**.





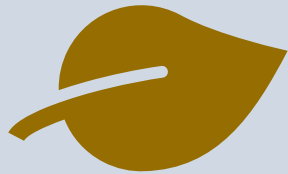
Overview

- I. Introduction – explanation of modes, apparatus
- II. Theoretical model – CFD model justification, model validation
- III. Mode categorization – dimensionless numbers, phase diagram
- IV. Mode properties – initial conditions, trajectory, landing positions
- V. Conclusions



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Motion of a leaf

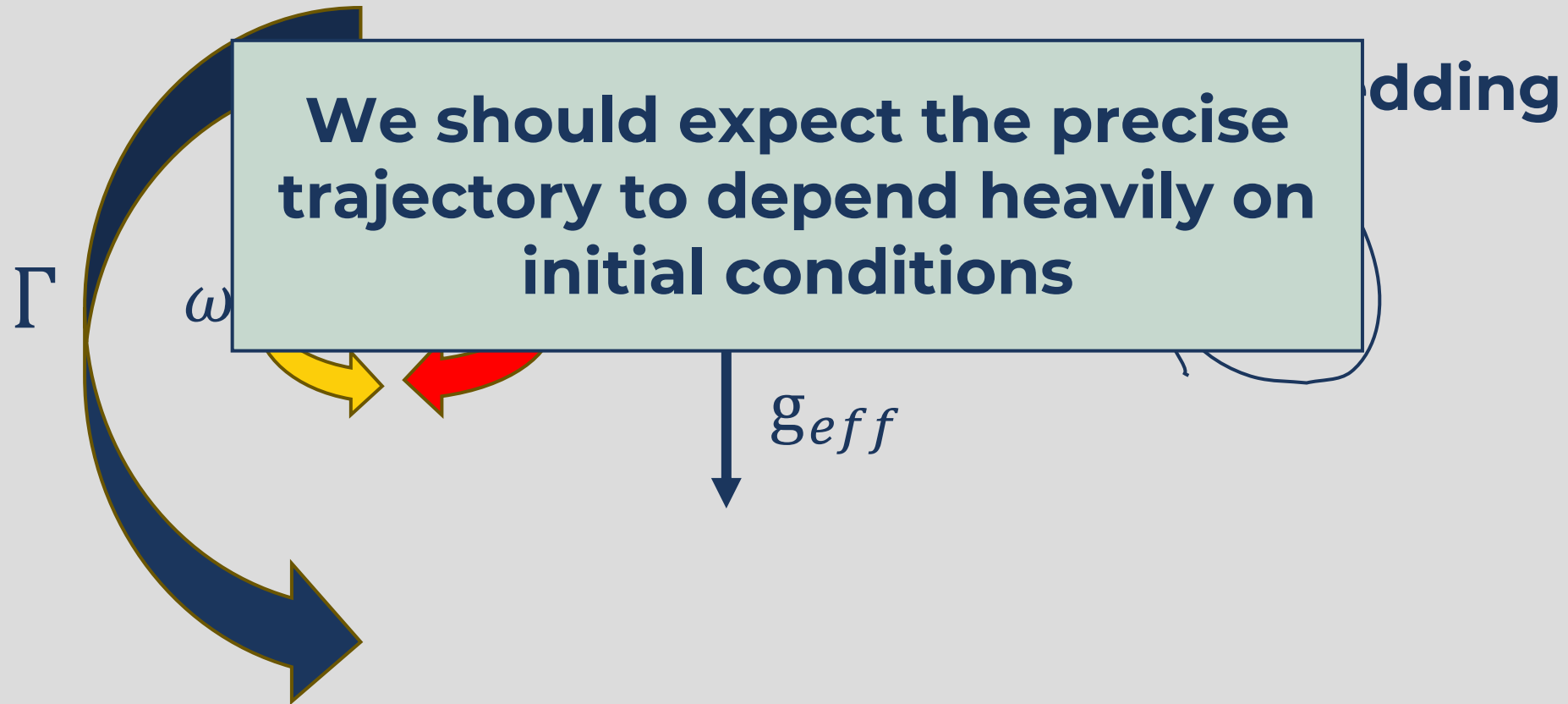


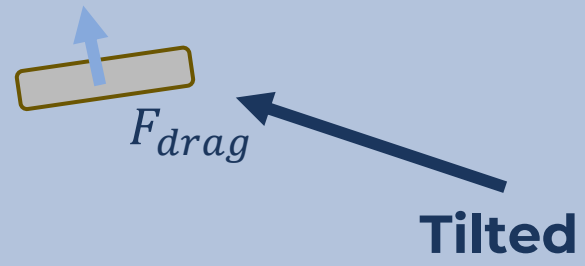
Perfectly straight

**We will focus mainly on
the other modes**

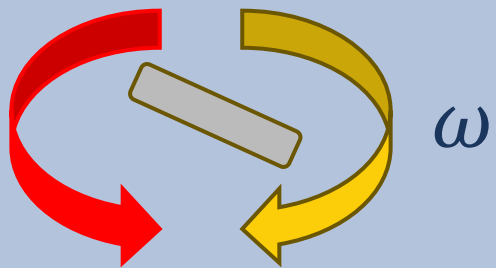


Thin coin $\Rightarrow$ more susceptible to instabilities



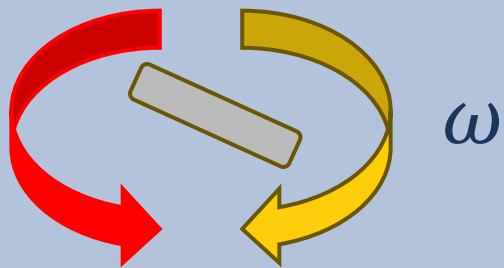


Water inertia



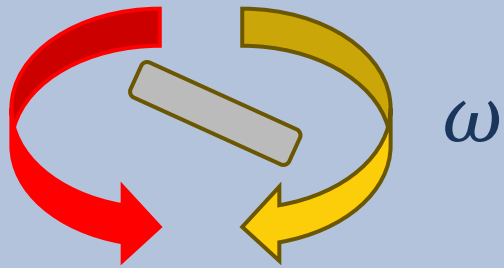
Fluttering

Water inertia



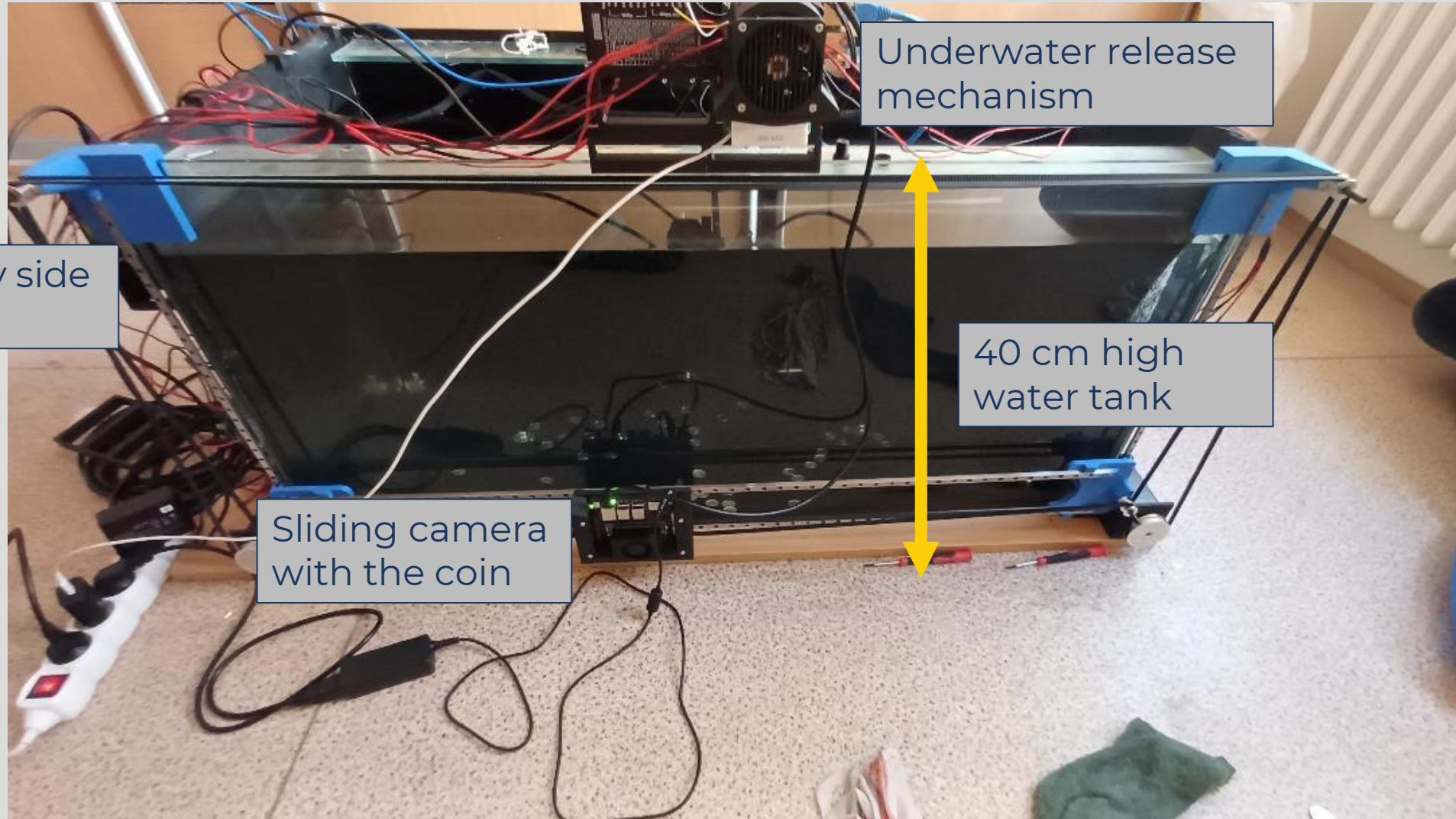
Tumbling

Water inertia



Chaotic

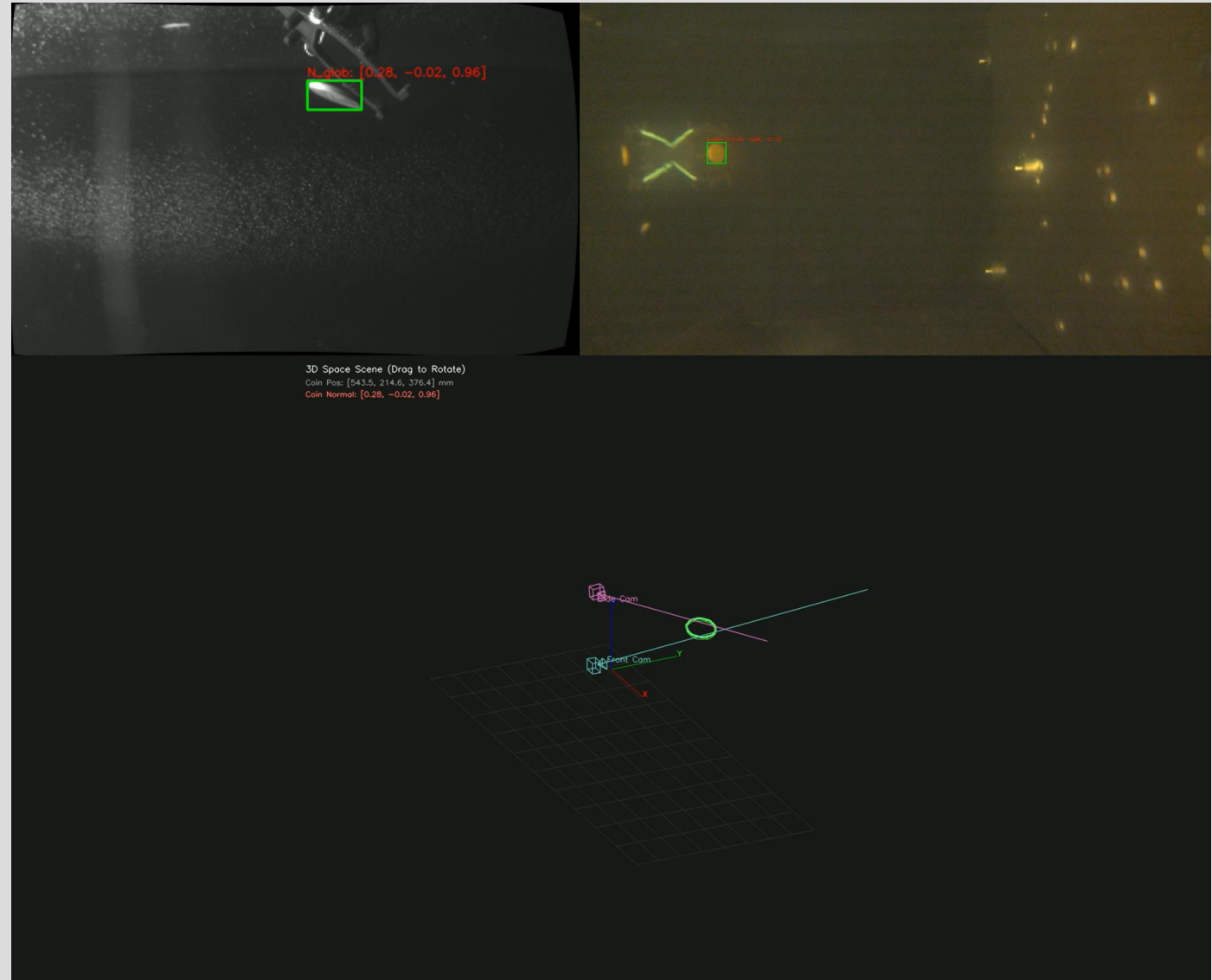
Apparatus



Apparatus



Methodology



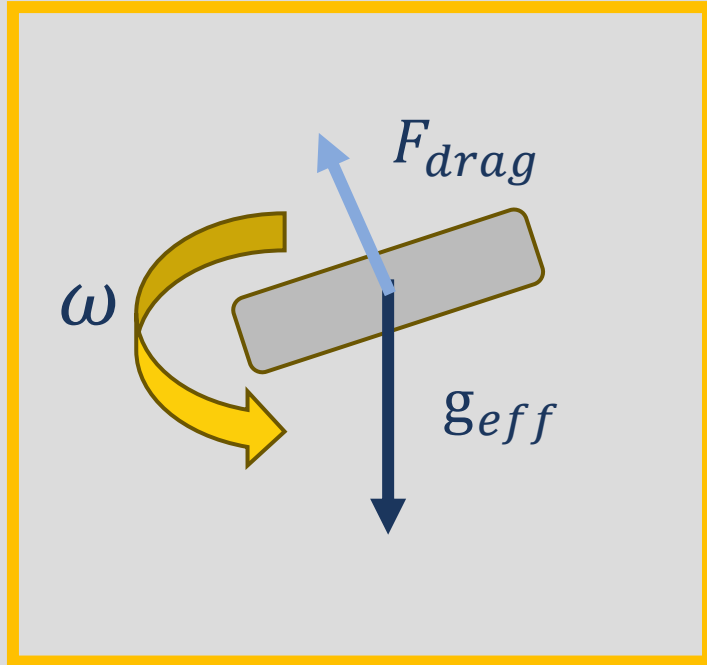


Overview

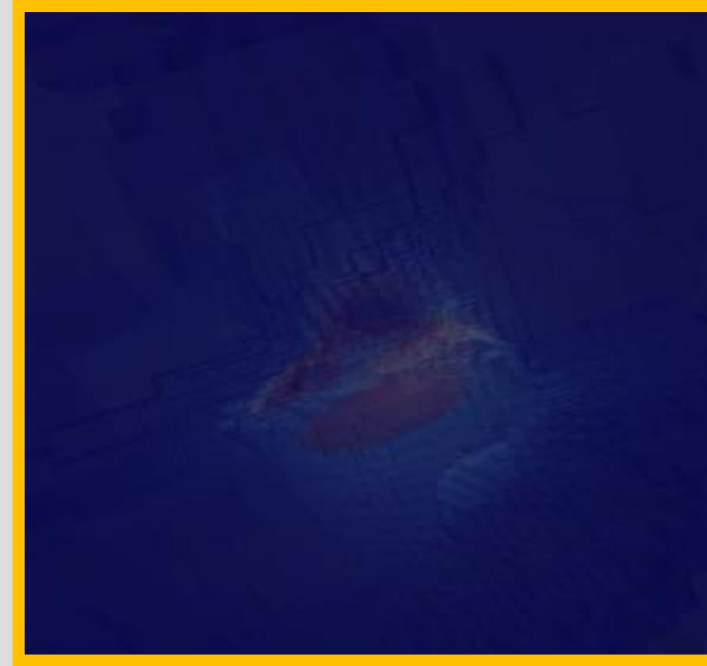
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Modelling the phenomenon

Dynamical model

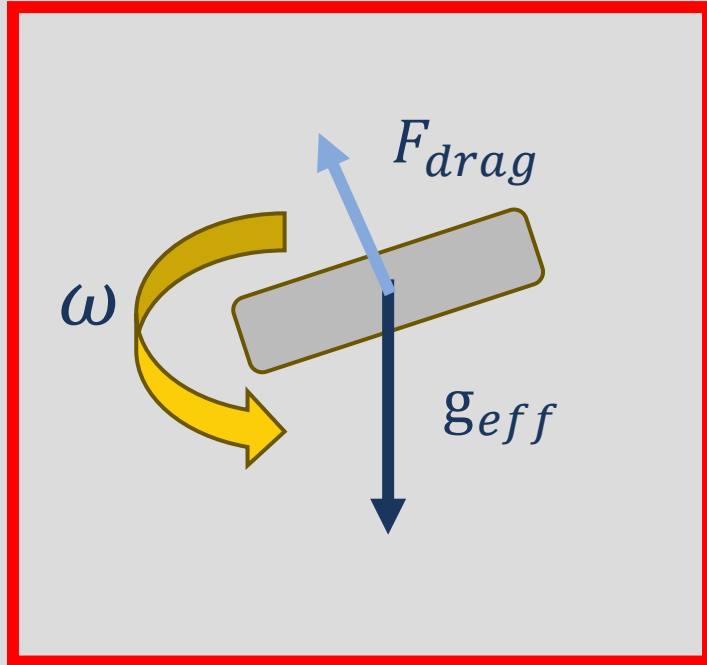


CFD model



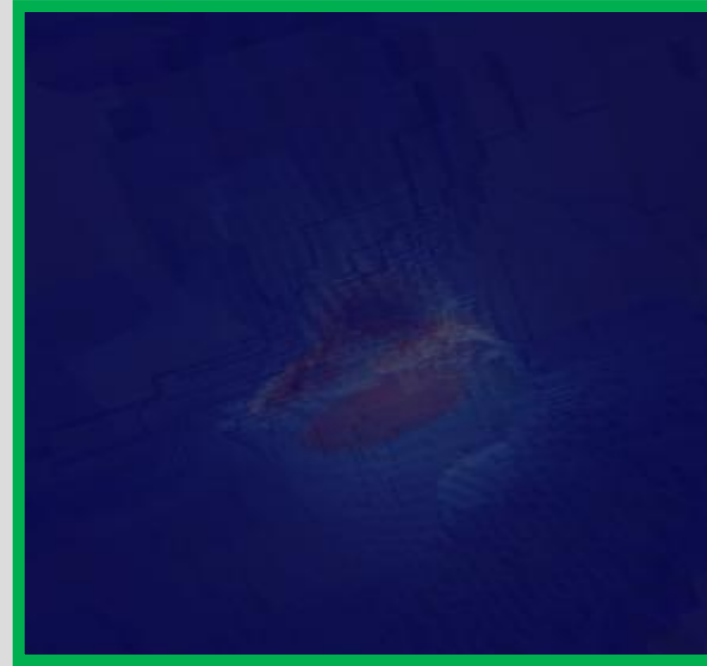
Modelling the phenomenon

Dynamical model



Doesnt account for
vortex shedding

CFD model



Incompressible Navier-Stokes equations

$$\nabla \cdot \mathbf{u} = 0$$

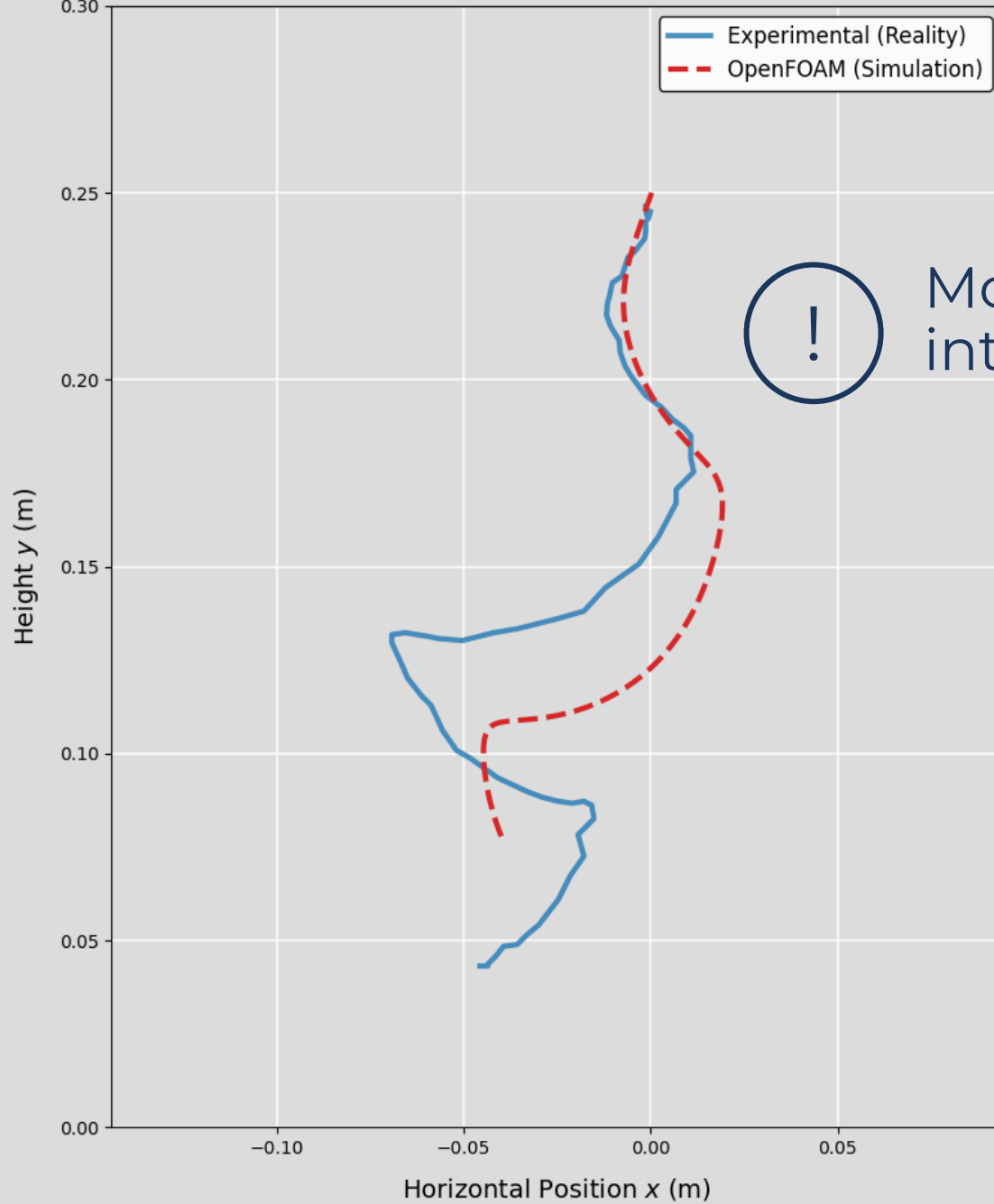
$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot [(\mathbf{u} - \mathbf{u}_m) \otimes \mathbf{u}] = -\nabla \left(\frac{p}{\rho} \right) + \nabla \cdot [\nu_{\text{eff}} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \mathbf{g}$$

Motion of the coin

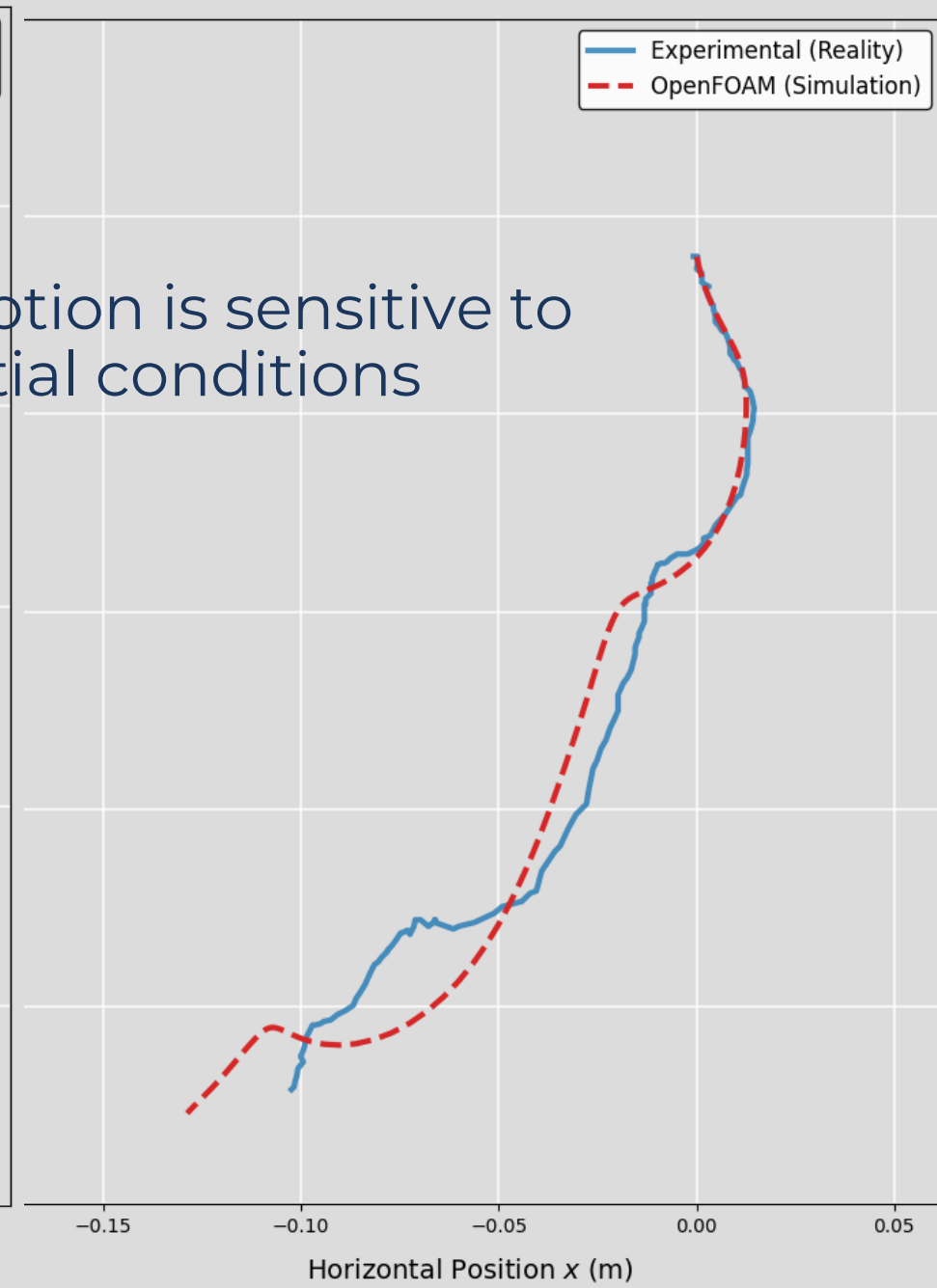
$$m \frac{d\mathbf{v}}{dt} = \mathbf{F}_{\text{fluid}} + m \mathbf{g}$$

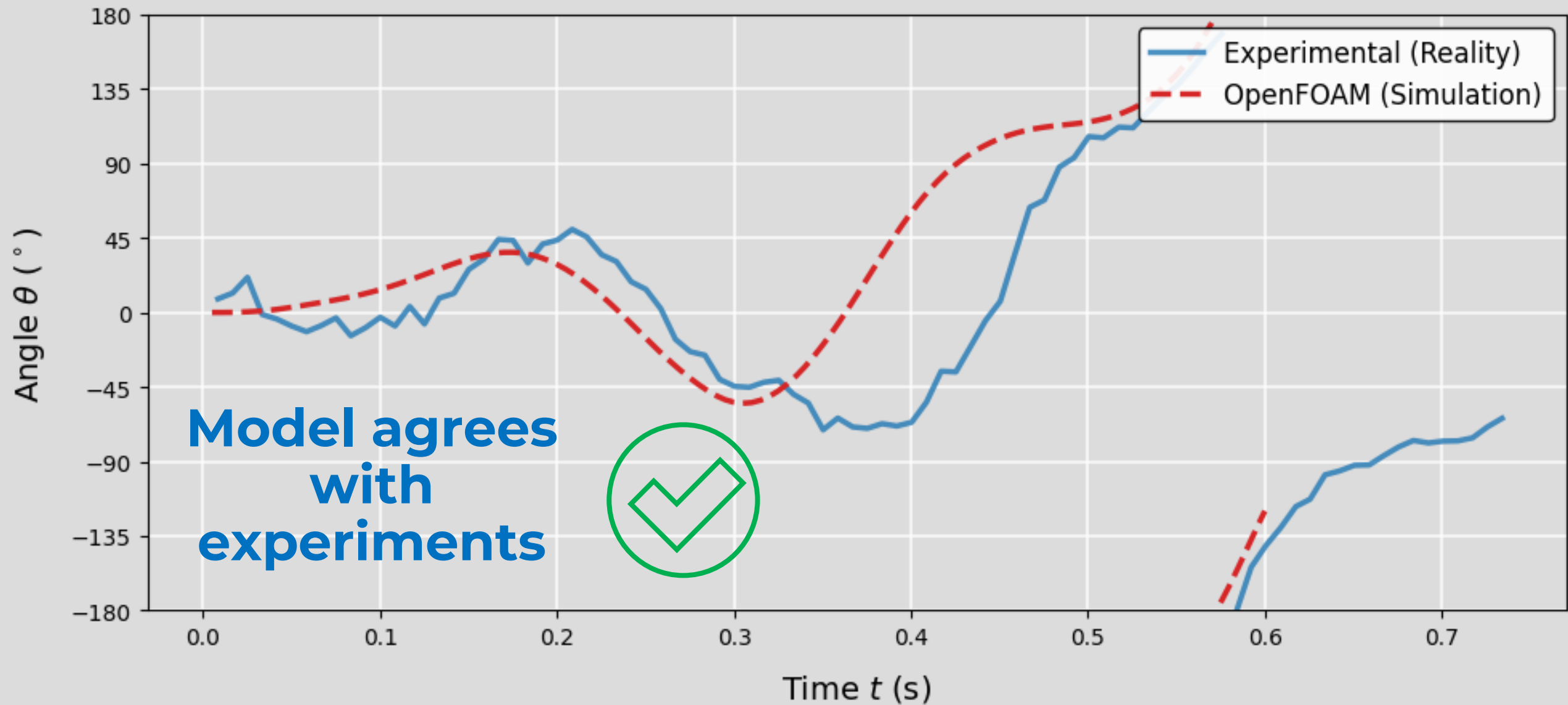
Rotation of the coin

$$\mathbf{I} \cdot \frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = \mathbf{M}_{\text{fluid}}$$



Motion is sensitive to initial conditions







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Incompressible Navier-Stokes equations

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot [(\mathbf{u} - \mathbf{u}_m) \otimes \mathbf{u}] = -\nabla \left(\frac{p}{\rho} \right) + \nabla \cdot [\nu_{\text{eff}} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \mathbf{g}$$

Motion of the coin

$$m \frac{d\mathbf{v}}{dt} = \mathbf{F}_{\text{fluid}} + m \mathbf{g}$$

Rotation of the coin

$$\mathbf{I} \cdot \frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = \mathbf{M}_{\text{fluid}}$$

Dimensionless numbers

Reynolds and Froudes number

$$Re = \frac{Ud}{\nu}$$

$$Fr = \frac{U}{\sqrt{gd}}$$



We can discard Froude's number since it doesn't affect the phenomenon

Dimensionless mass

$$m^* = \frac{\pi}{4} \left(\frac{\rho_s}{\rho} \right) \left(\frac{t}{d} \right)$$

Dimensionless moment of inertia

$$I^* = \frac{I}{\rho d^5}$$

Dimensionless numbers

Reynolds number

$$Re = \frac{Ud}{\nu}$$



Dimensionless numbers help us find phases of the phenomenon

Dimensionless mass

$$m^* = \frac{\pi}{4} \left(\frac{\rho_s}{\rho} \right) \left(\frac{t}{d} \right)$$

Dimensionless moment of inertia

$$I^* = \frac{I}{\rho d^5}$$

Dimensionless numbers

Reynolds number

$$Re = \frac{Ud}{\nu}$$

d Coin diameter

U Average coin vertical velocity

ν Kinematic viscosity of water

t Coin thickness

ρ_s Coin material density

ρ Water density

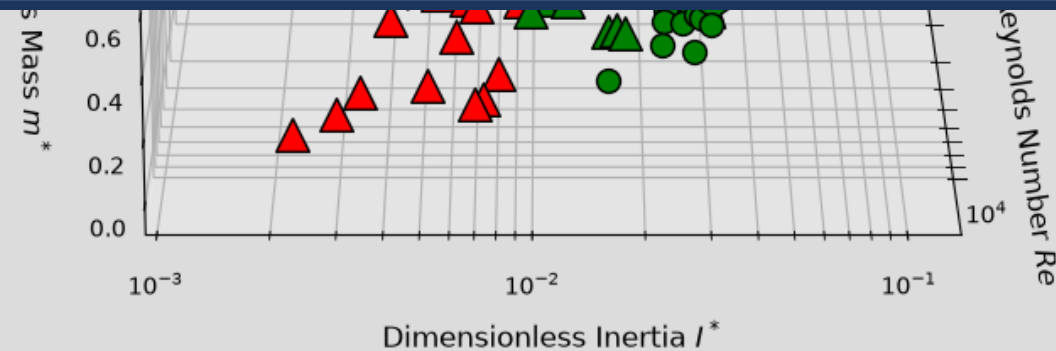
Dimensionless mass

$$m^* = \frac{\pi}{4} \left(\frac{\rho_s}{\rho} \right) \left(\frac{t}{d} \right)$$

Dimensionless moment of inertia

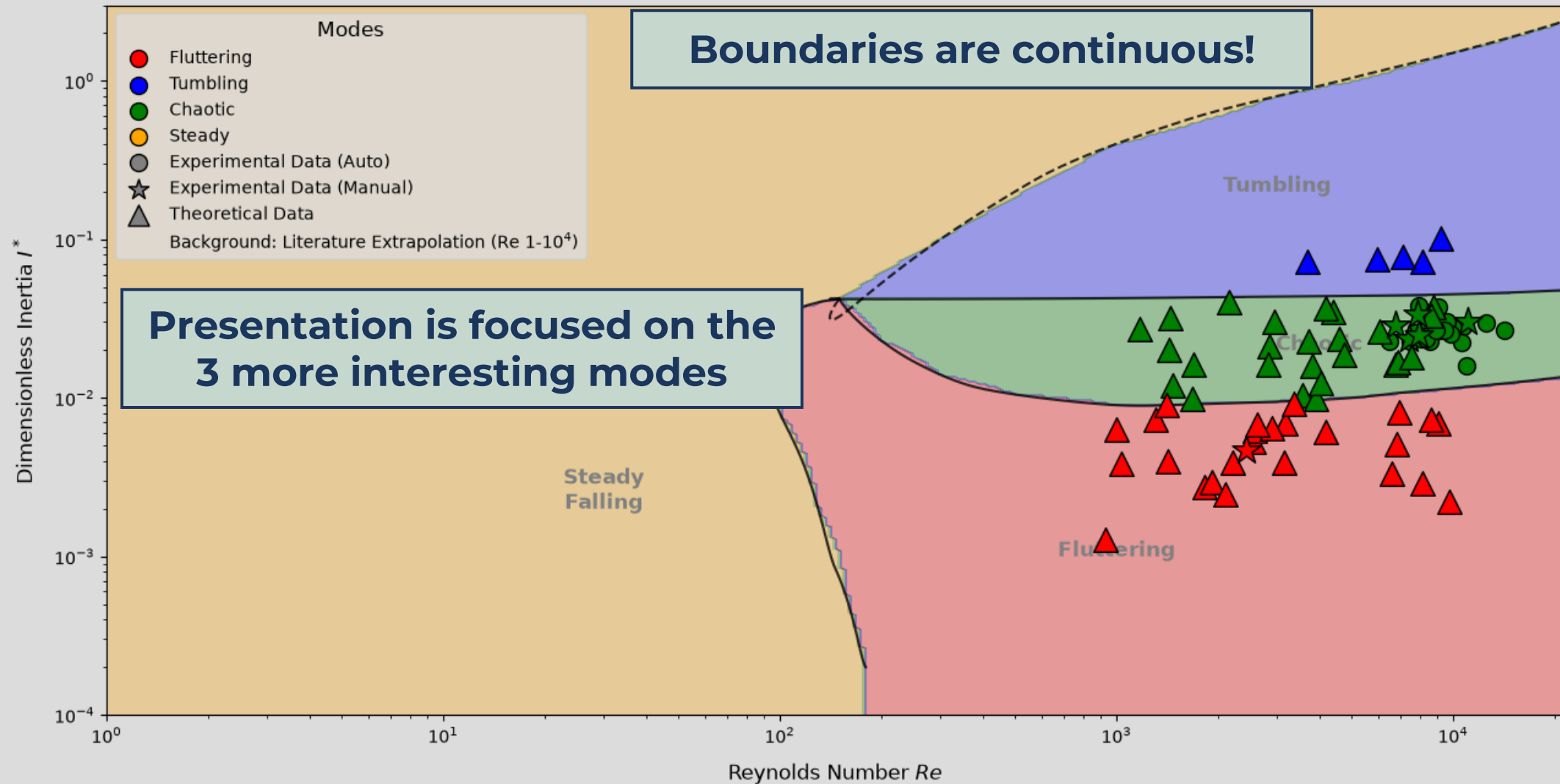
$$I^* = \frac{I}{\rho d^5}$$

Lets categorize based on literature and previous description!



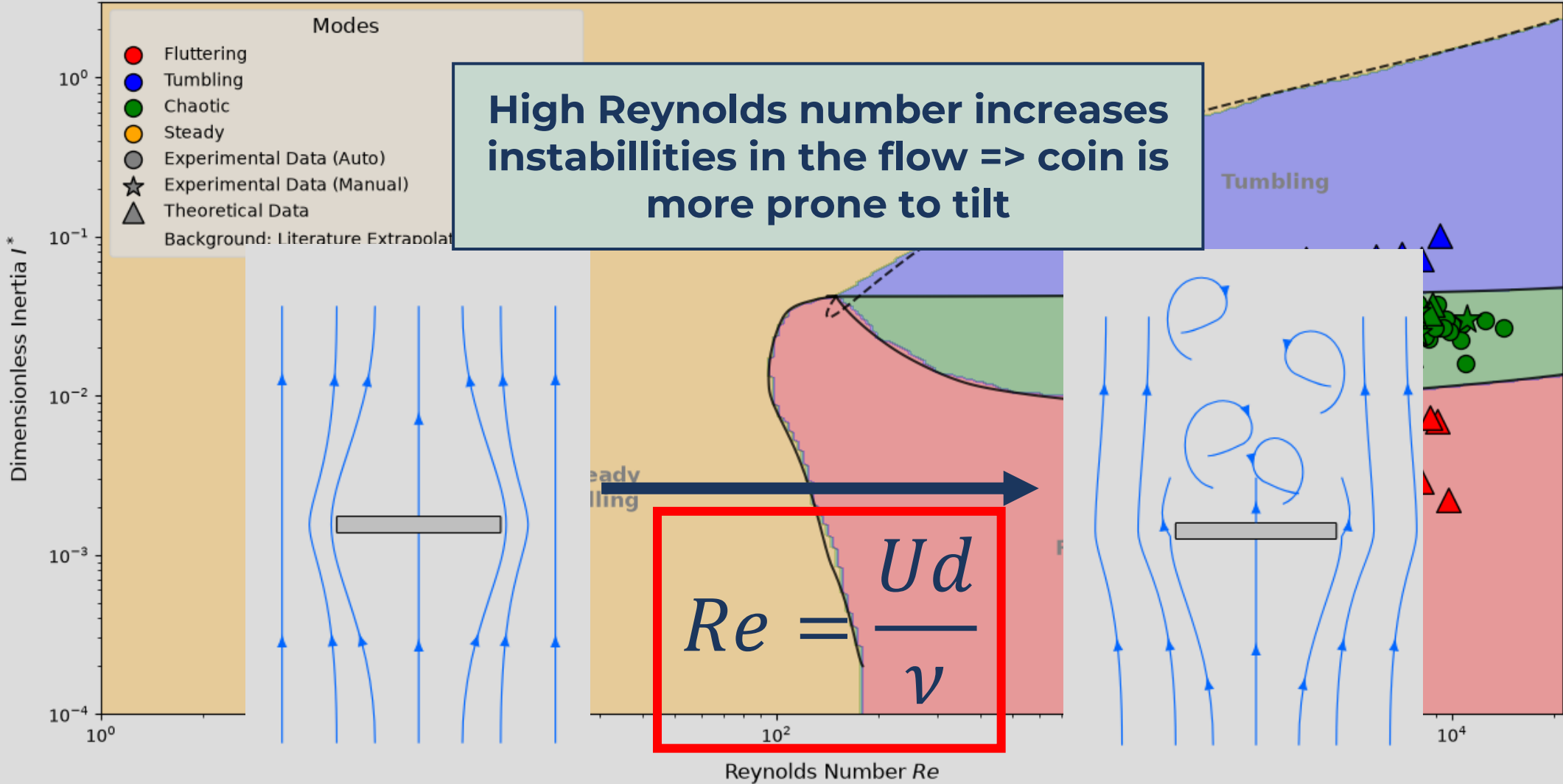
Luke Heisinger, Paul Newton and Eva Kanso. (2014)

2D Phase Diagram (Thin Coins: $t/d < 0.1$)

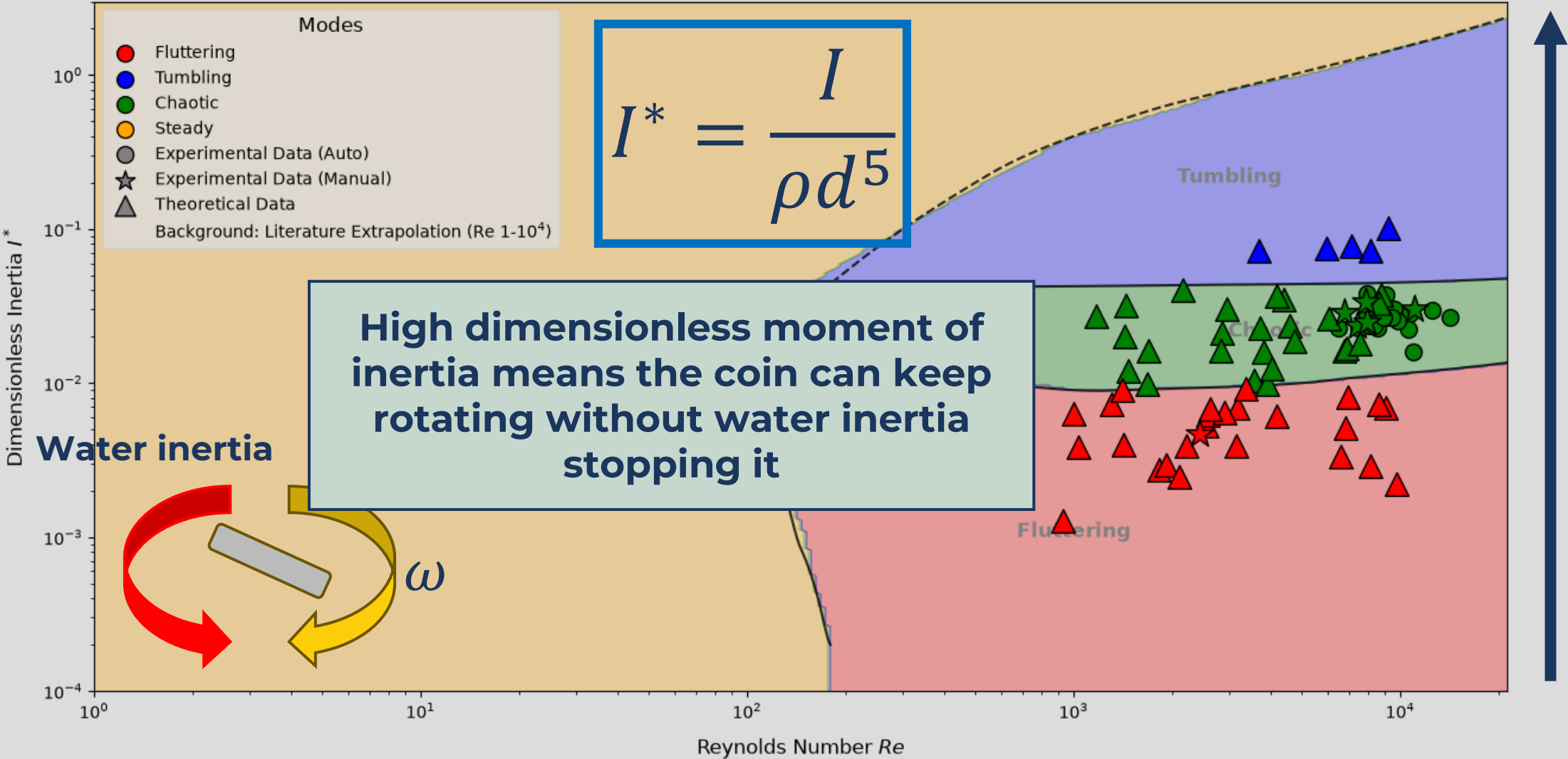


Luke Heisinger, Paul Newton and Eva Kanso. (2014)

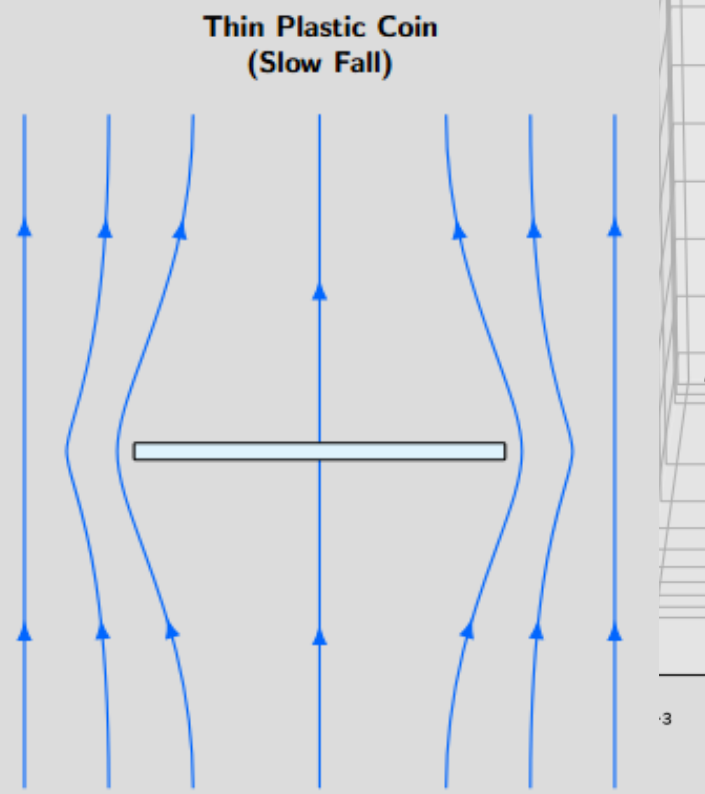
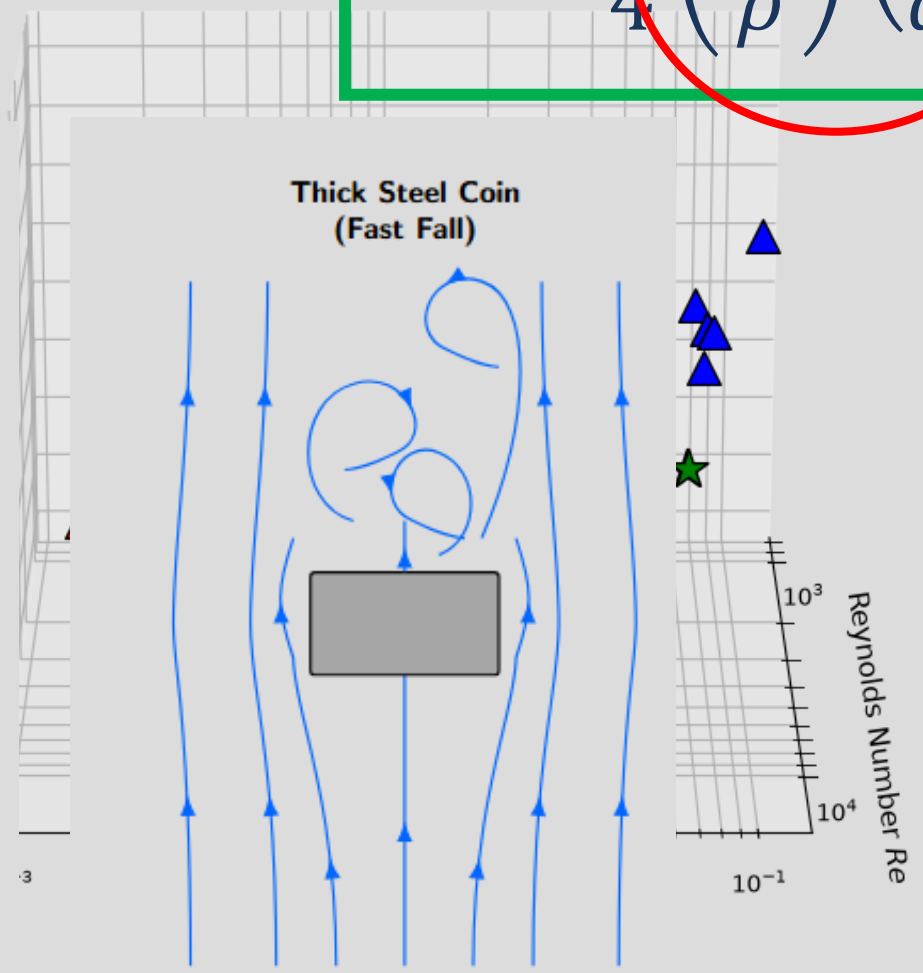
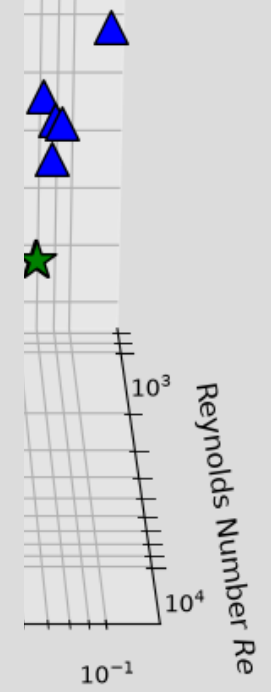
2D Phase Diagram (Thin Coins: $t/d < 0.1$)



2D Phase Diagram (Thin Coins: $t/d < 0.1$)

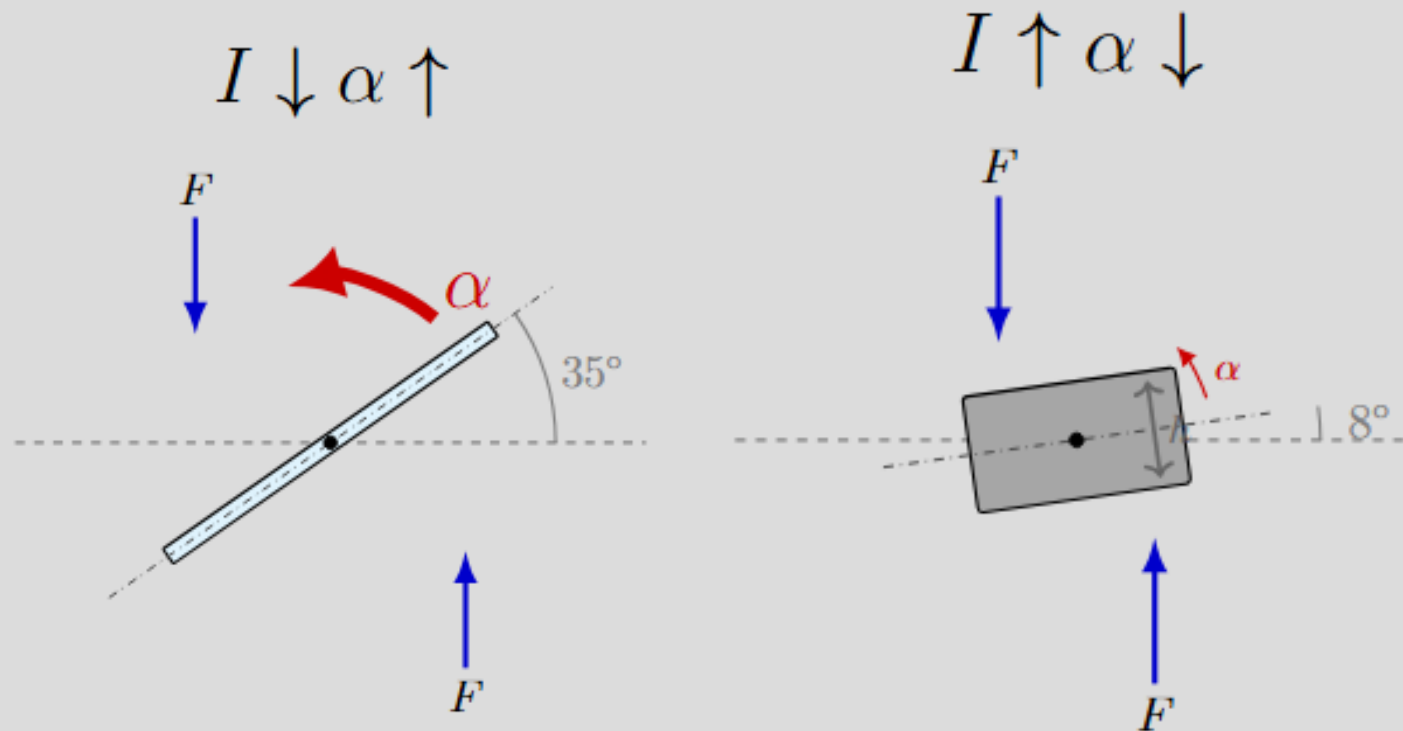


$$m^* = \frac{\pi}{4} \left(\frac{\rho_s}{\rho} \right) \left(\frac{t}{d} \right)$$



- Modes**
- Fluttering
 - Tumbling
 - Chaotic
 - Steady
 - Experimental Data (○)
 - ★ Experimental Data (★)
 - ▲ Theoretical Data

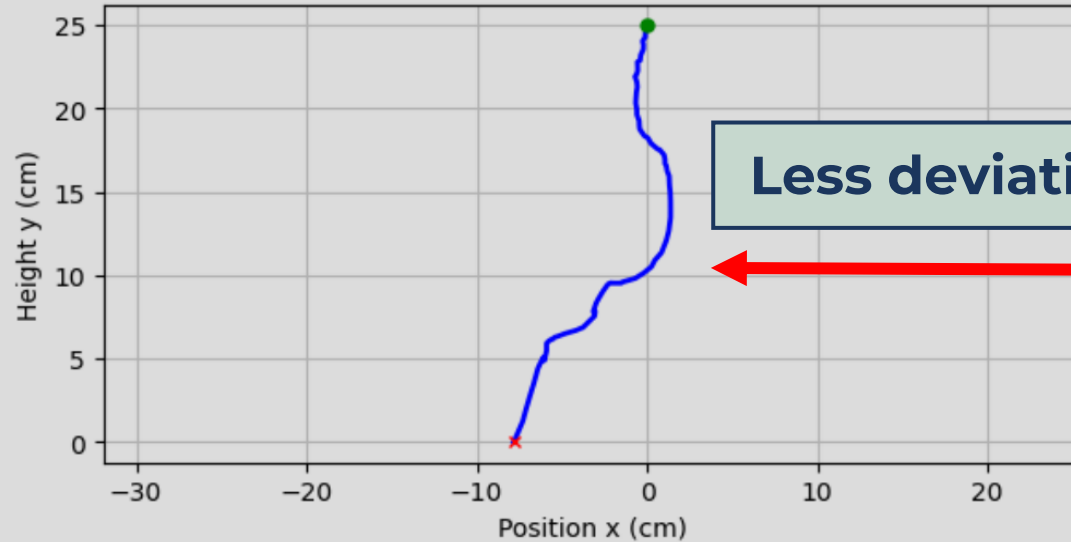
$$m^* = \frac{\pi}{4} \left(\frac{\rho_s}{\rho} \right) \left(\frac{t}{d} \right)$$



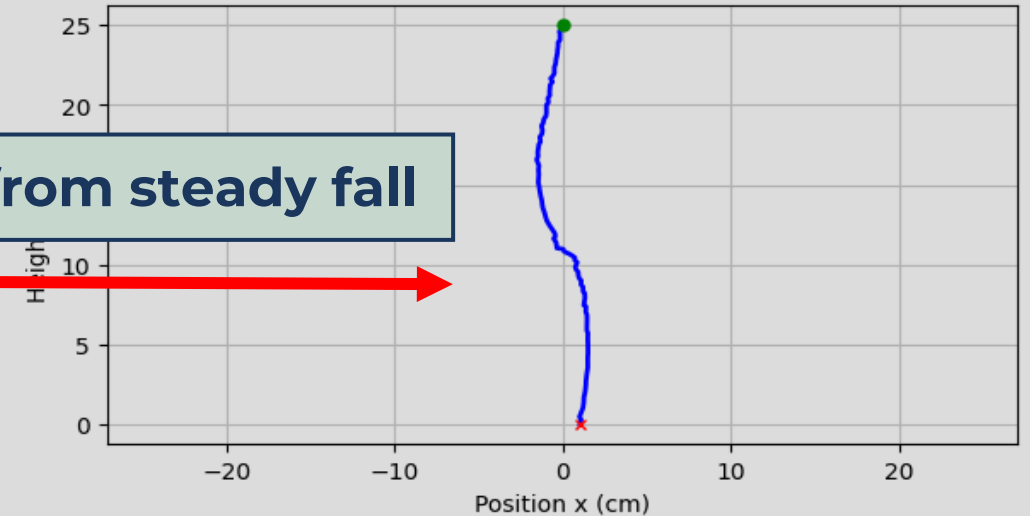
High dimensionless mass means the coin falls faster and its harder to tilt

Effective mass impact

$$U = 34 \text{ cm/s}$$



$$U = 50 \text{ cm/s}$$



Less deviation from steady fall

Effective mass correlates to velocity

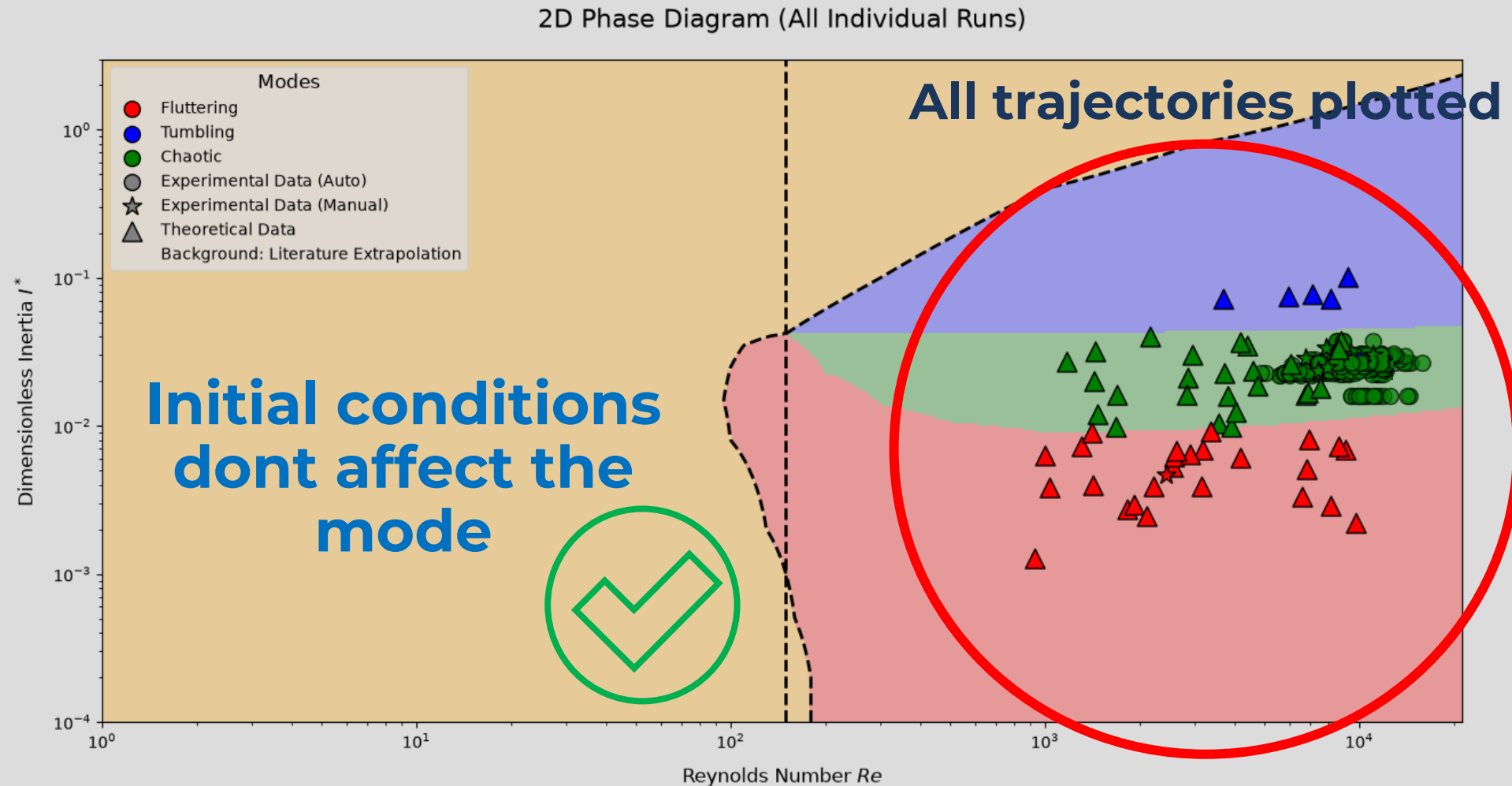




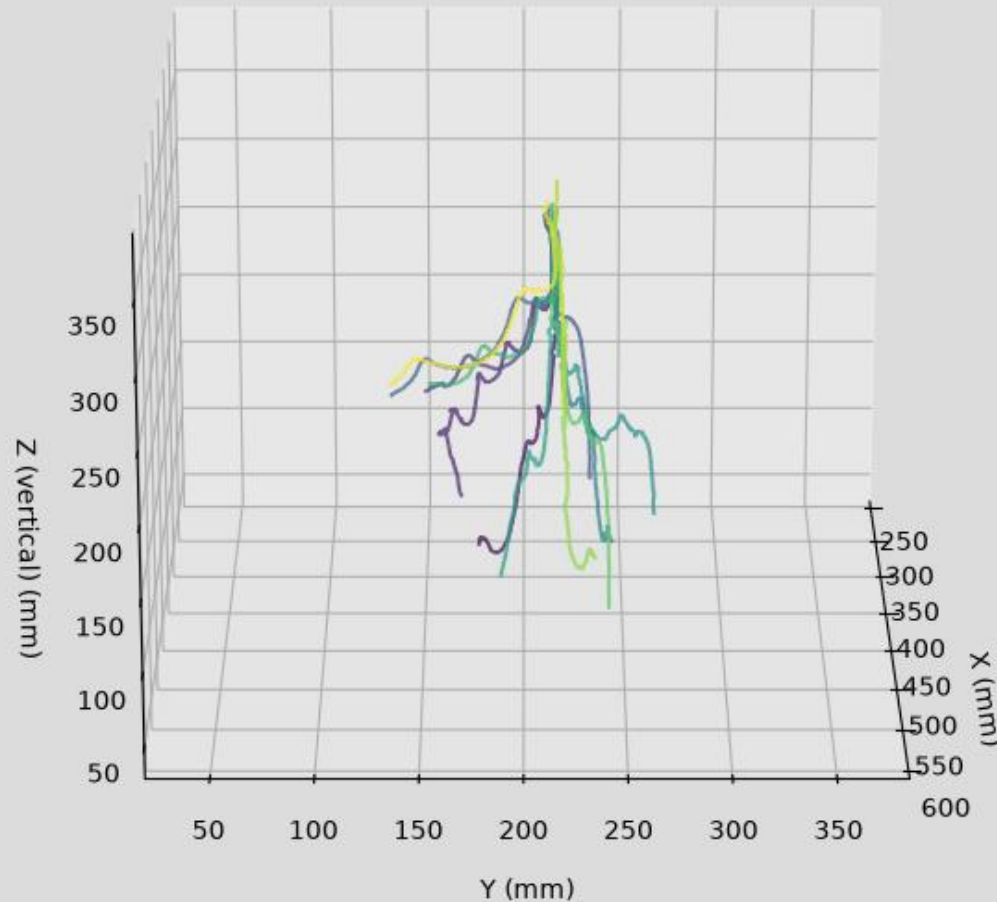
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Initial conditions



Sensitivity to initial conditions

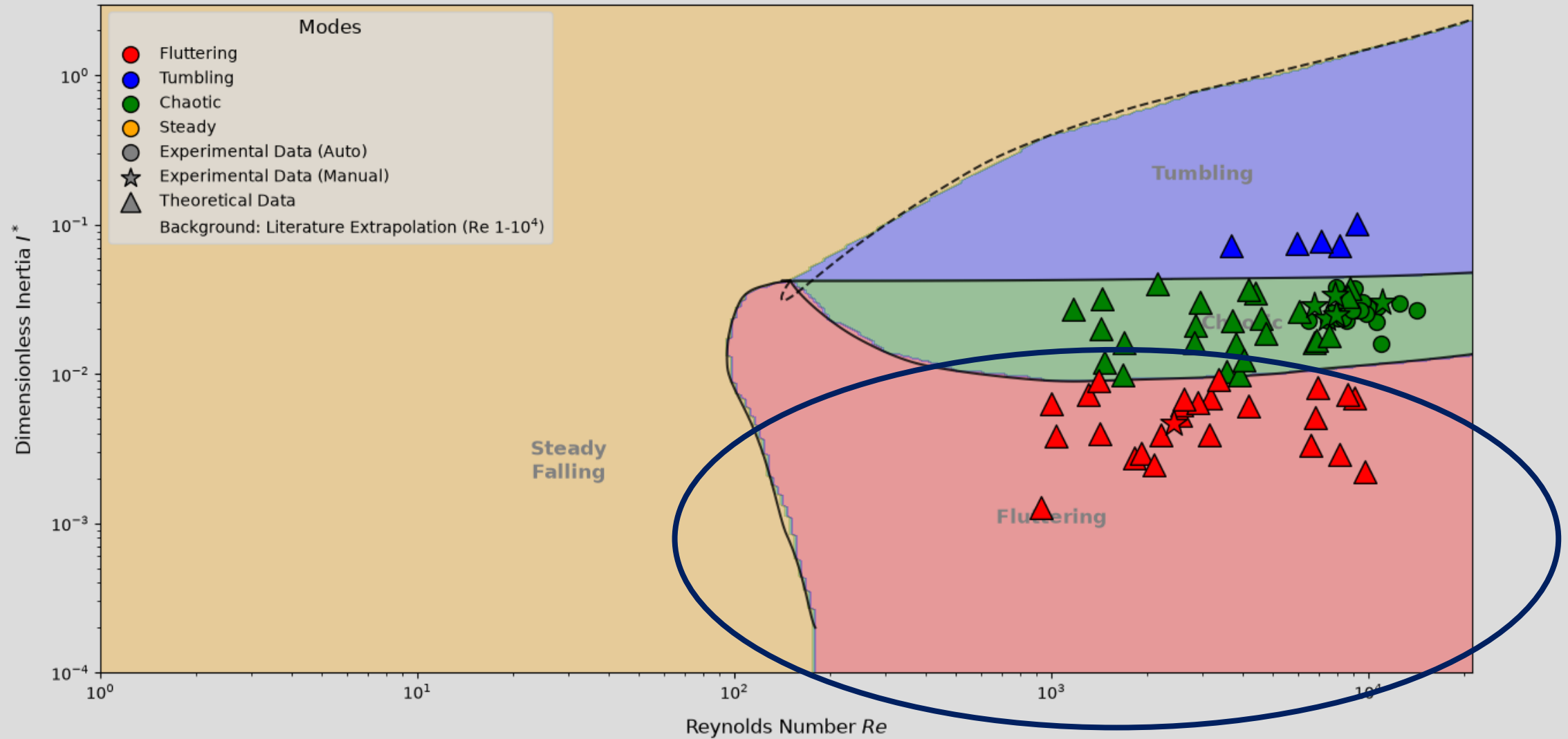


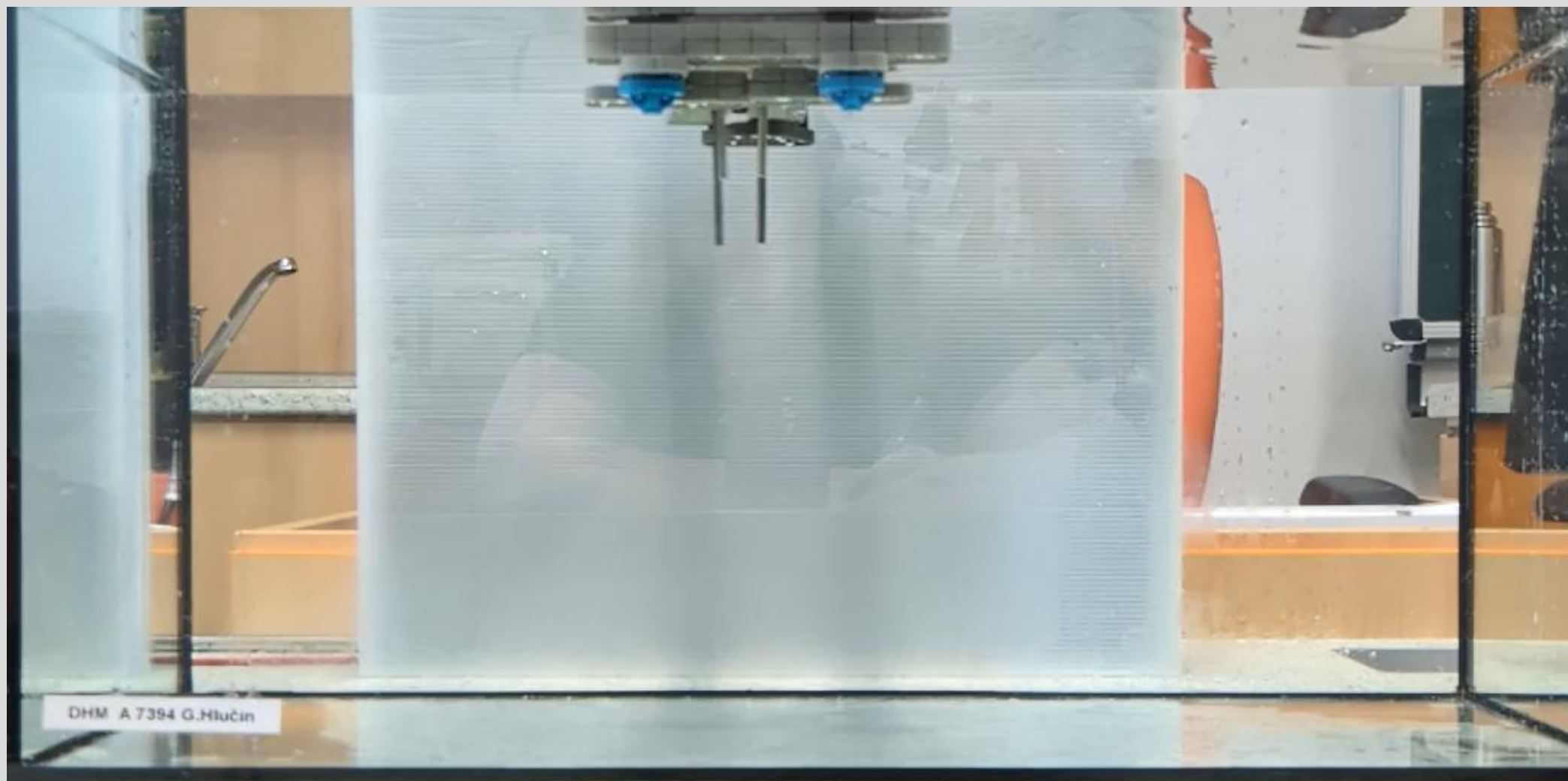
Almost the same initial condition leads to vastly different trajectories



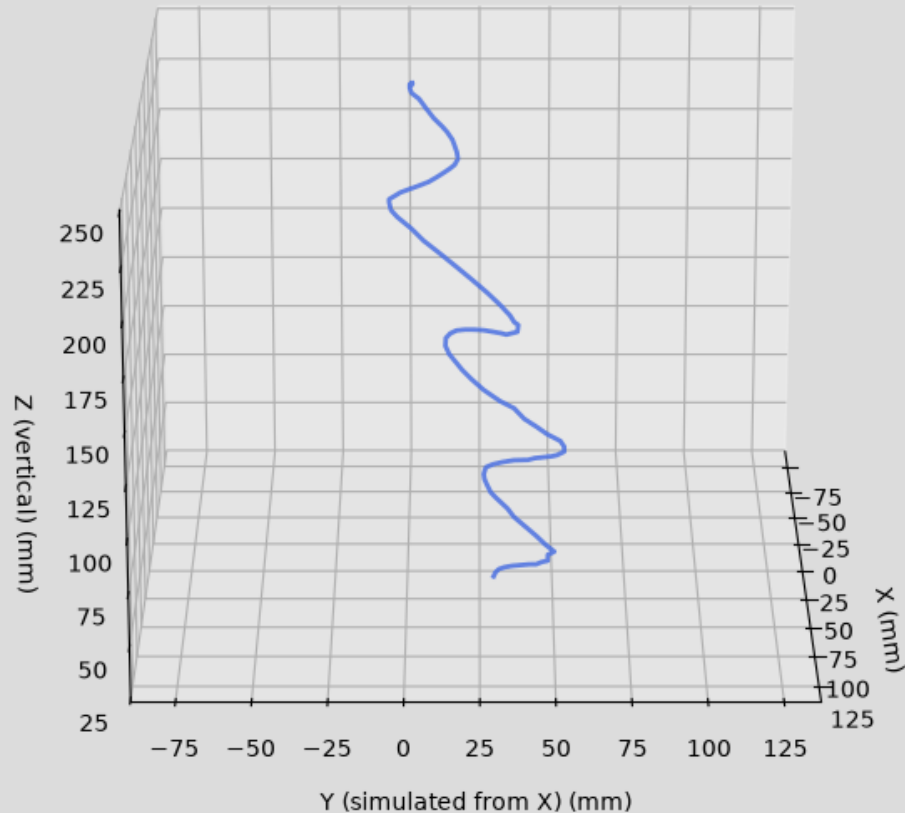
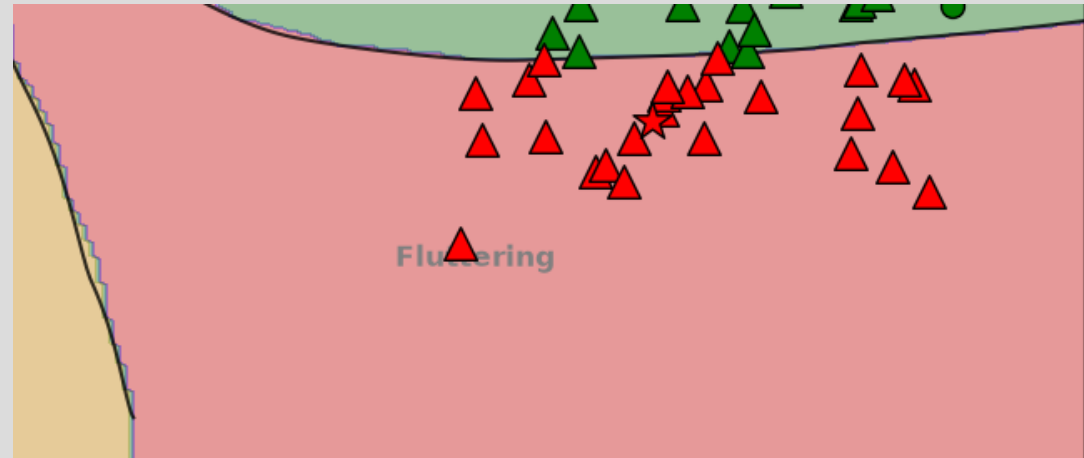
From experiments the Lyapunov exponent will be calculated to see how sensitive each mode is

2D Phase Diagram (Thin Coins: $t/d < 0.1$)



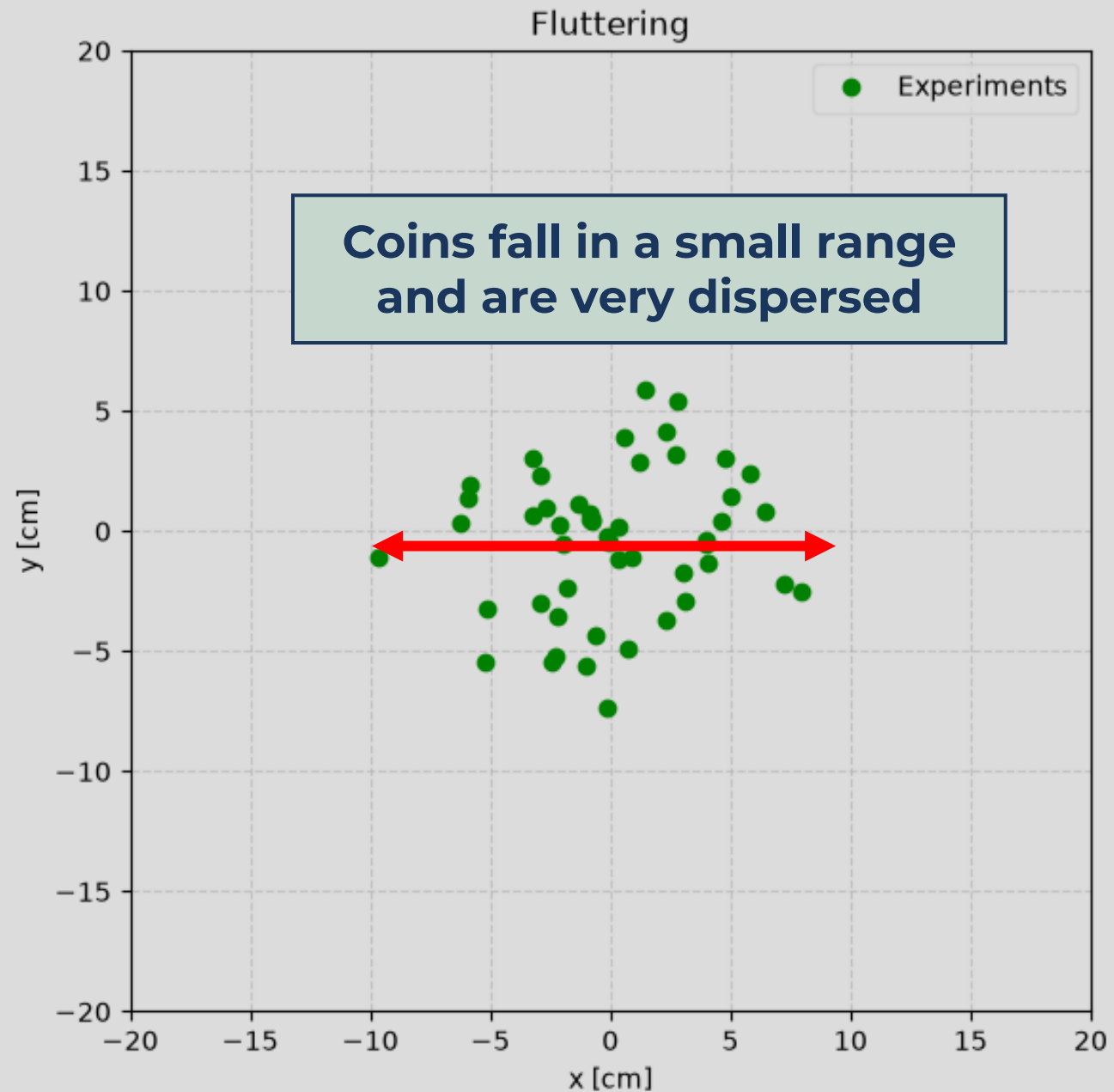


Fluttering

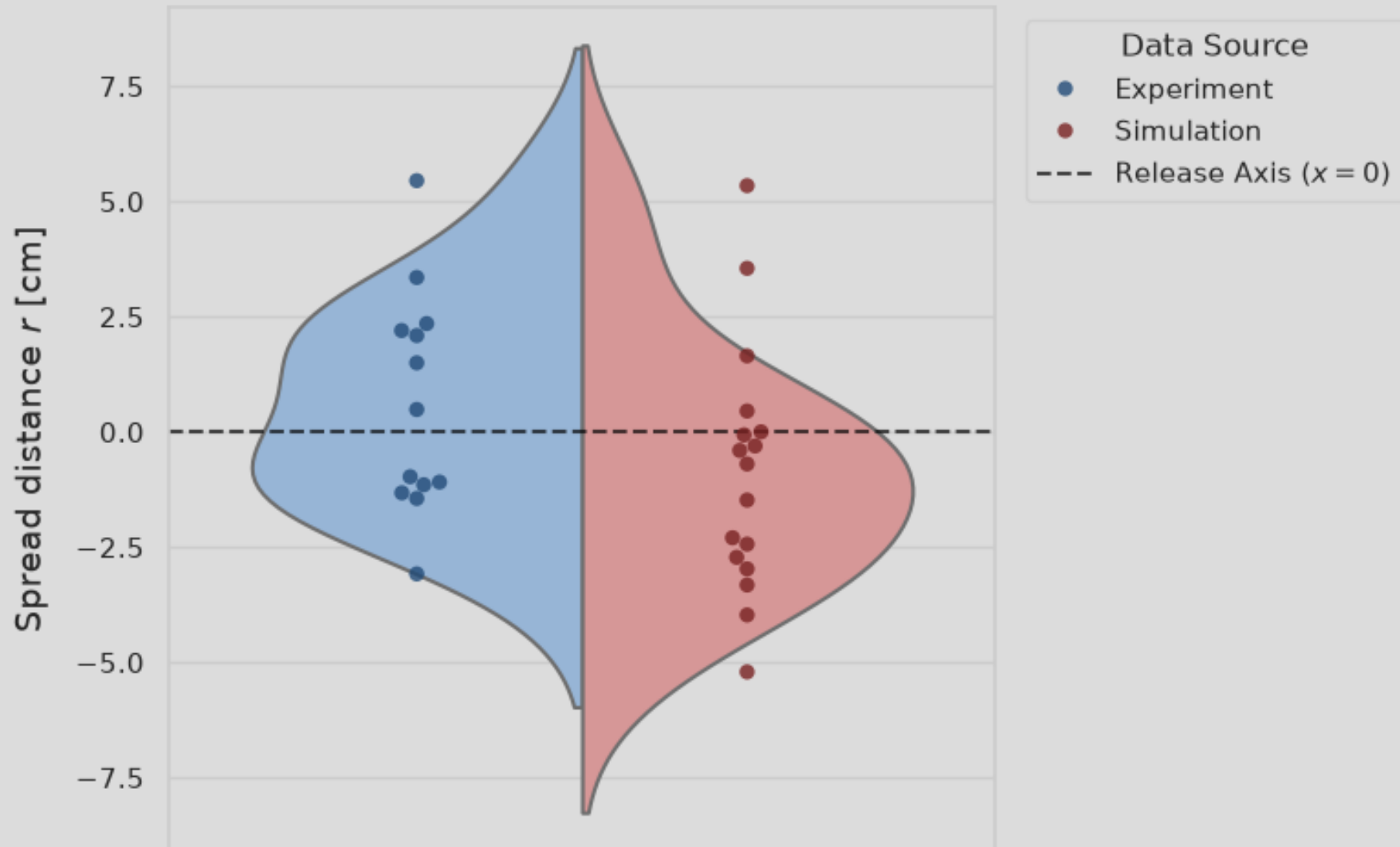


- Periodic motion
- Slower descend
- Lyapunov exponent:
 $\lambda = 1.2 \pm 0.2$

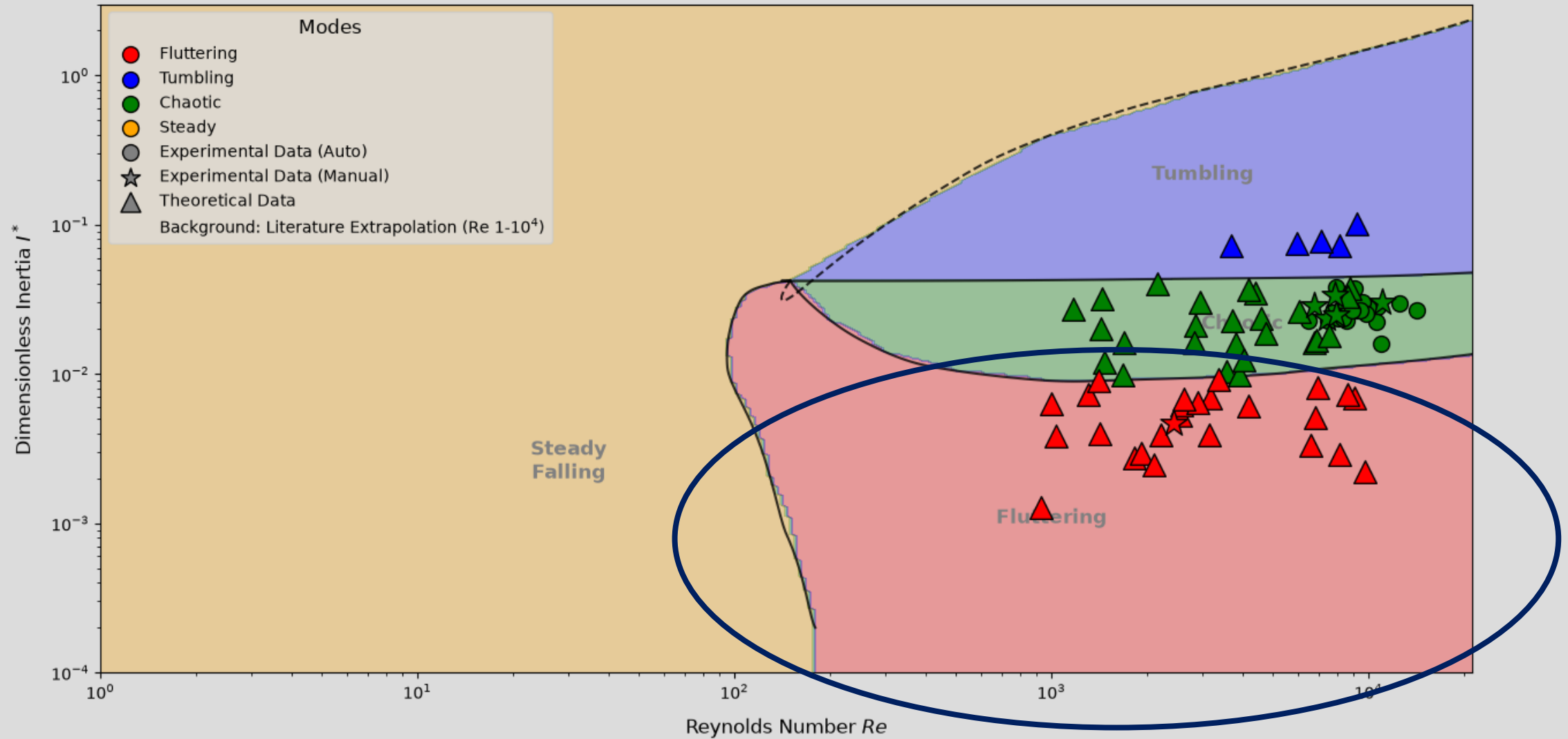
Landing positions



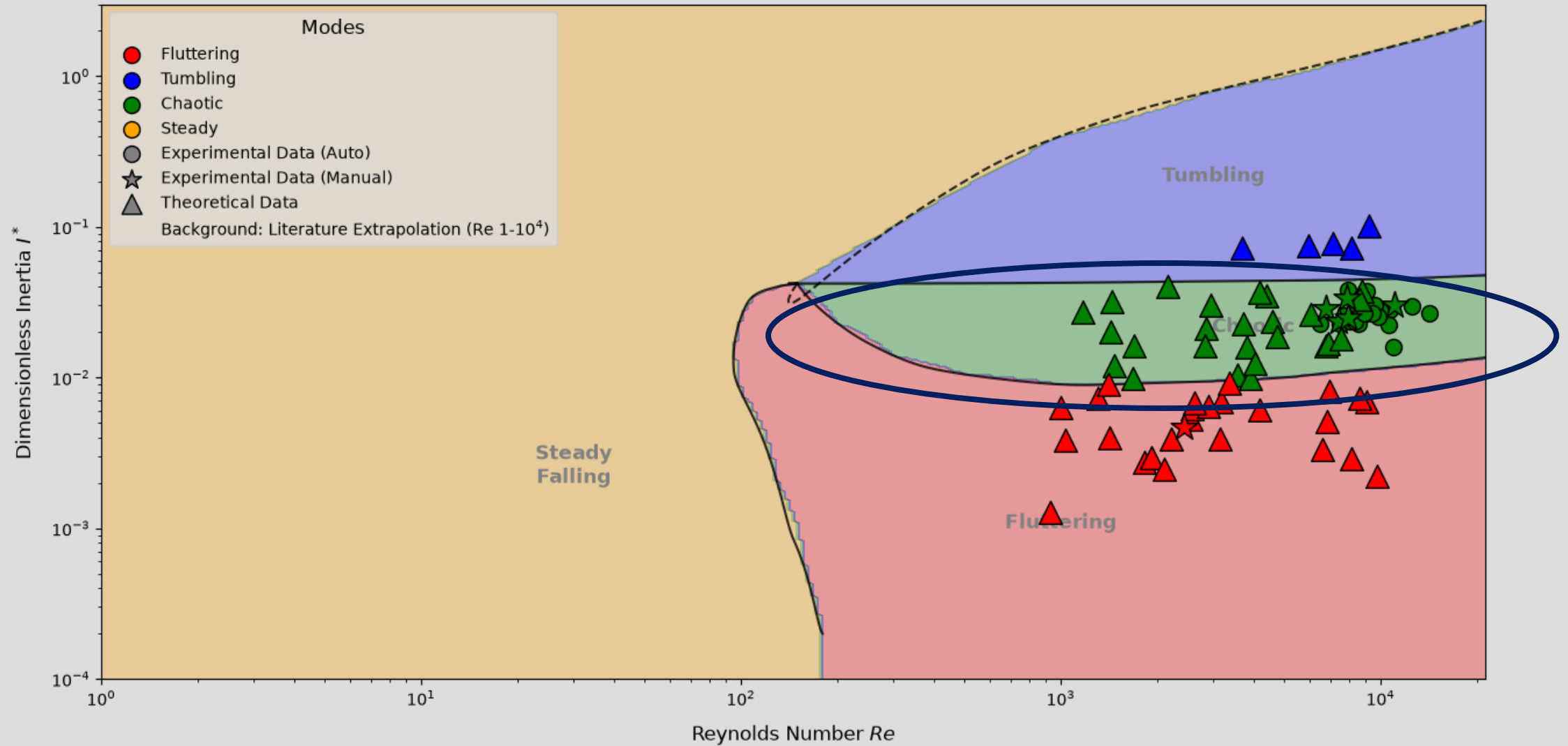
Dispersion prediction

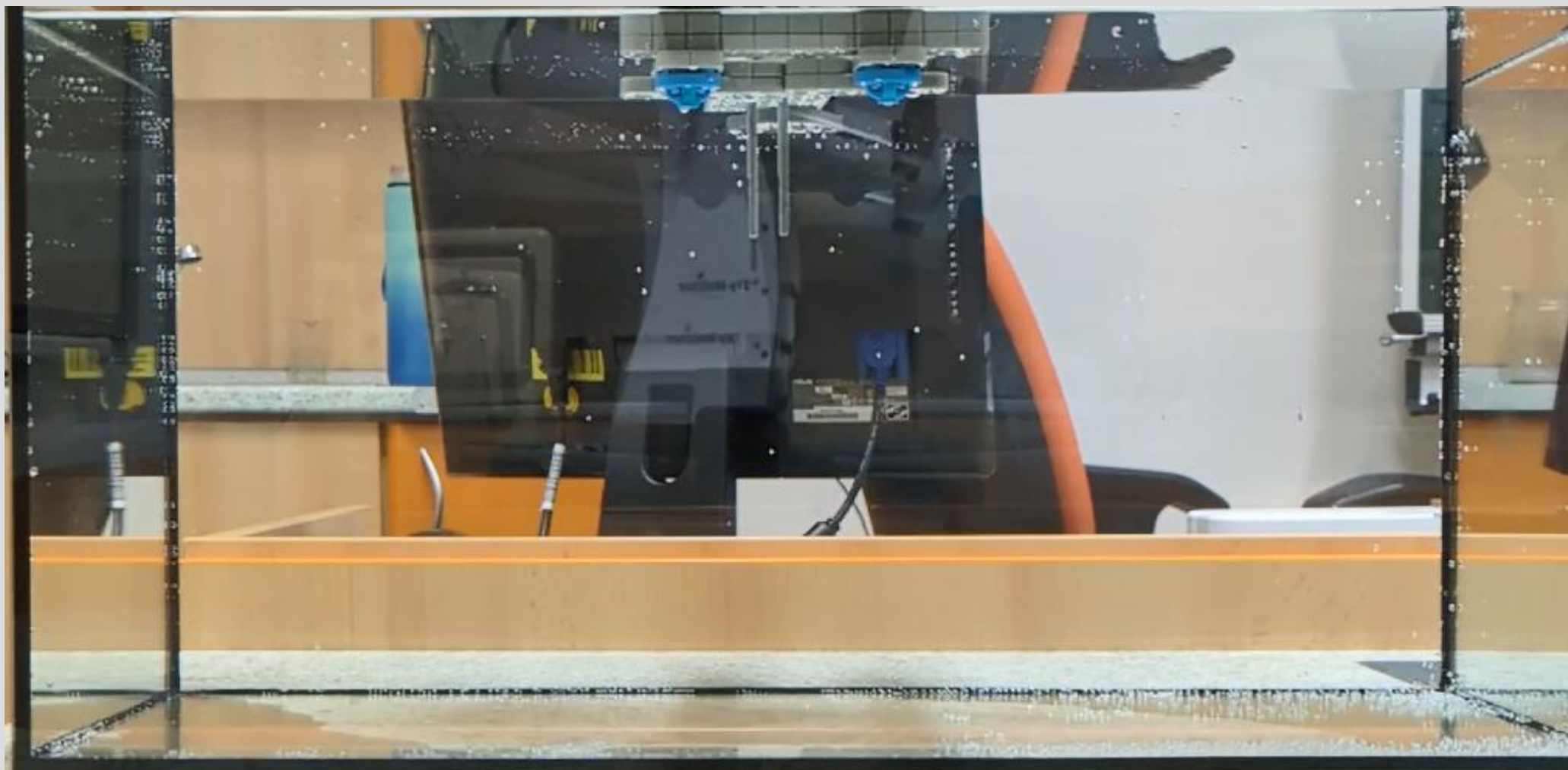


2D Phase Diagram (Thin Coins: $t/d < 0.1$)

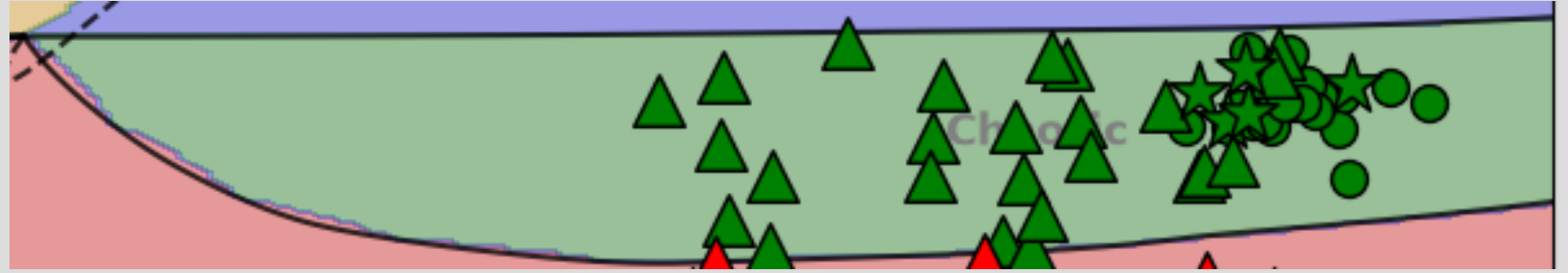
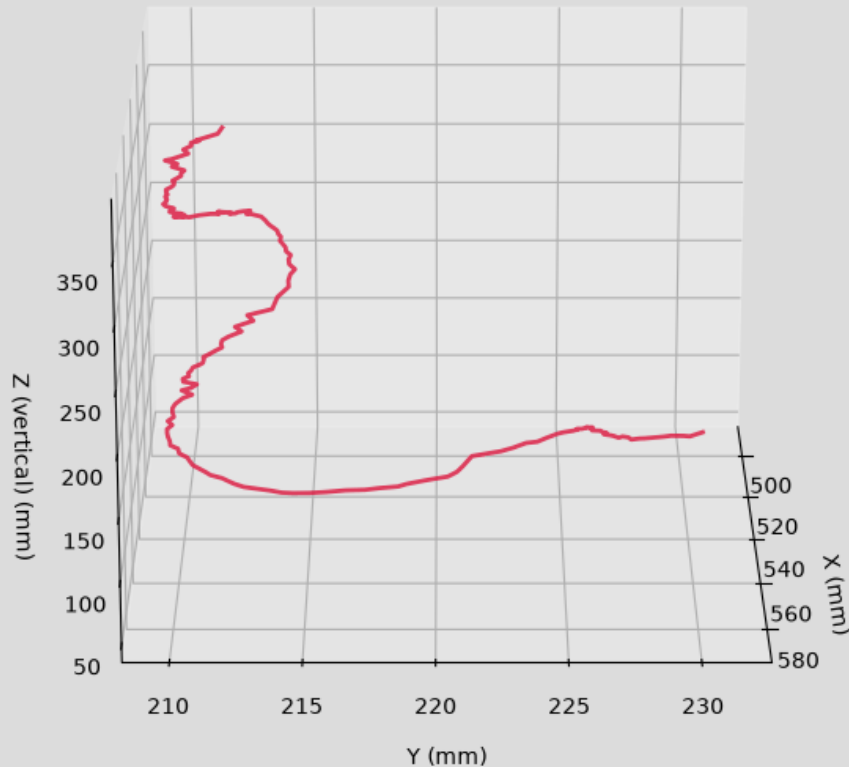


2D Phase Diagram (Thin Coins: $t/d < 0.1$)





Chaotic

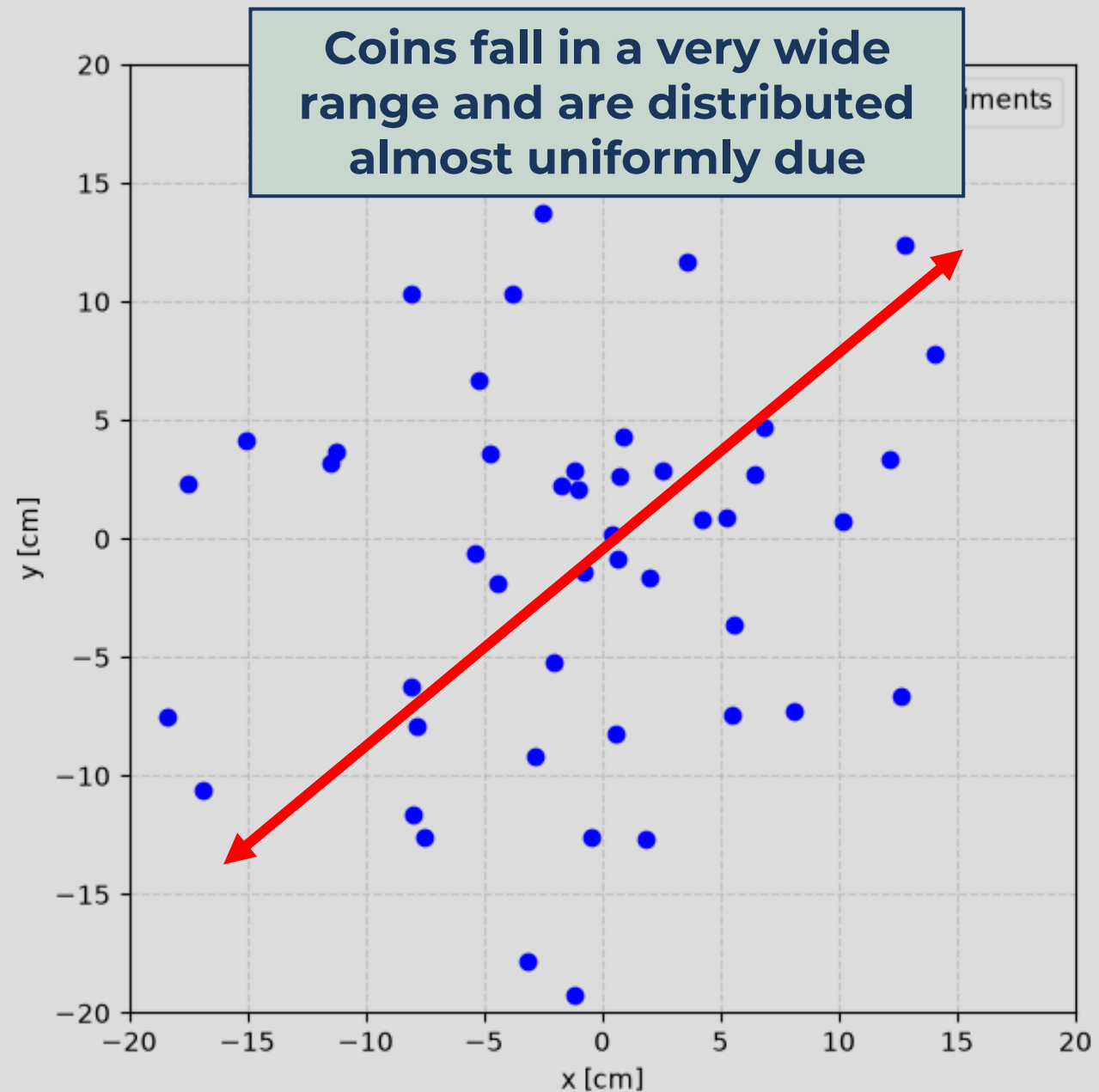


- Periodic motion
- Autorotation
- Higher sensitivity to initial conditions
- Lyapunov exponent:

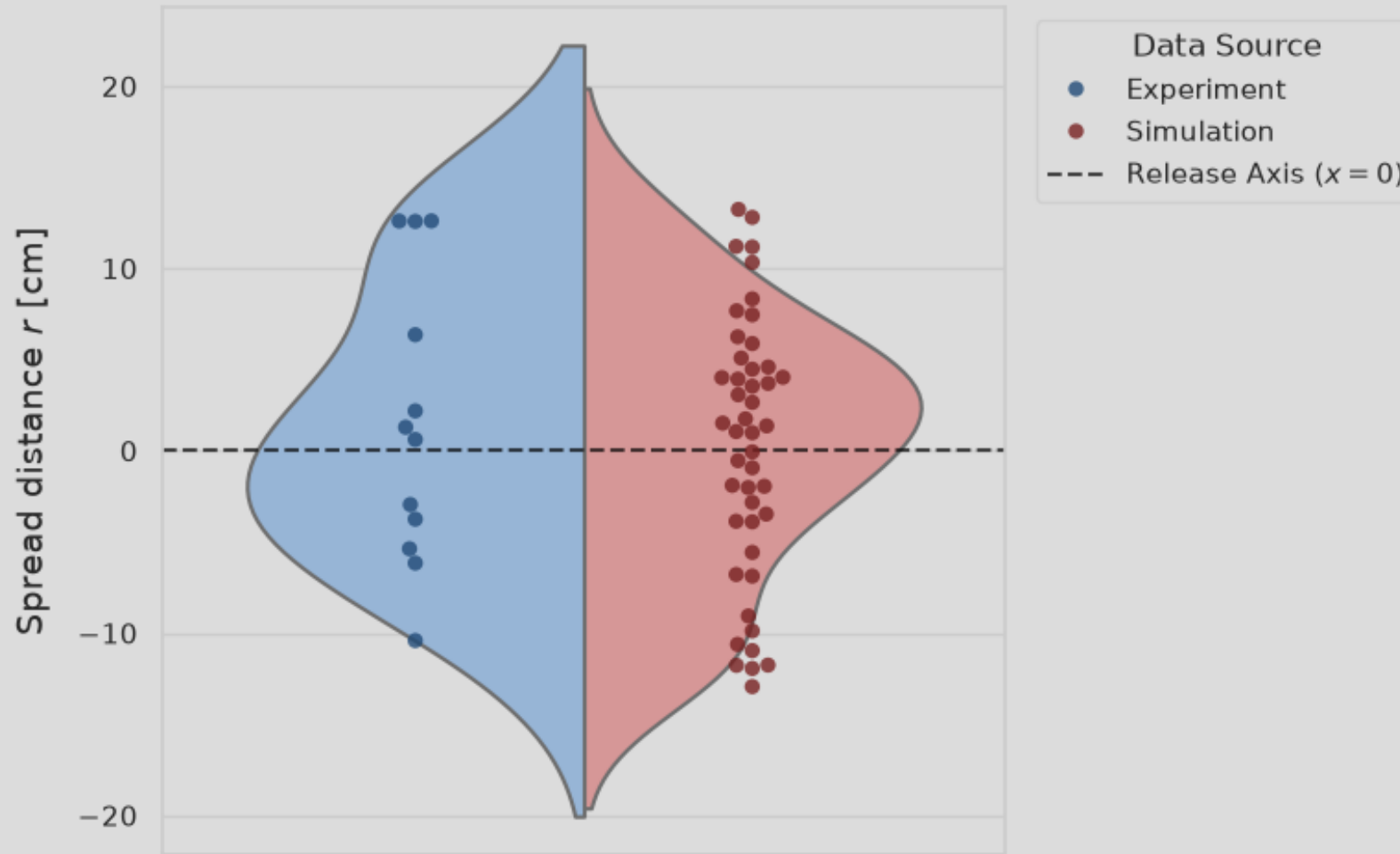
$$\lambda = 4.3 \pm 0.2$$

**The chaotic experiences
largest chaos**

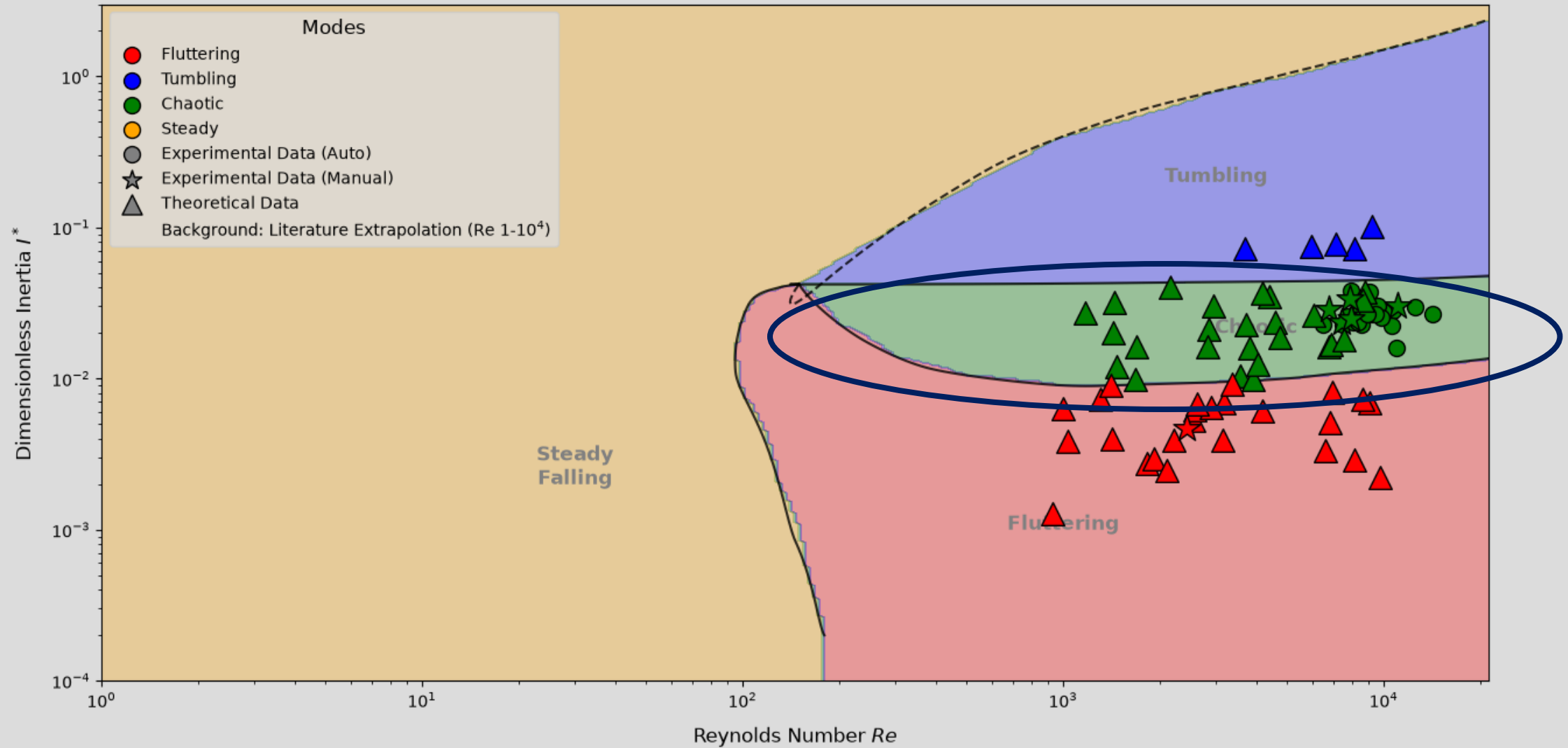
Dispersion



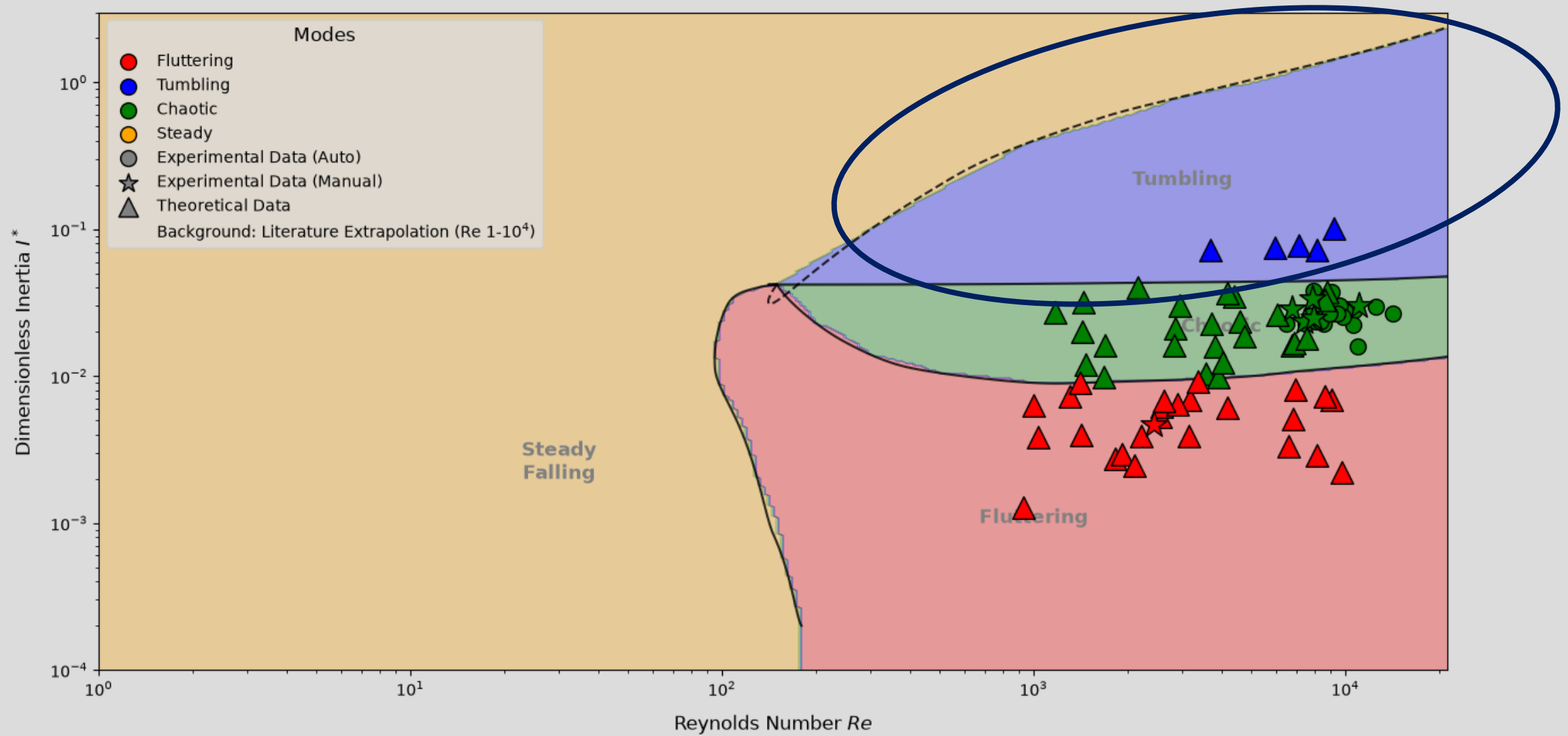
Dispersion prediction

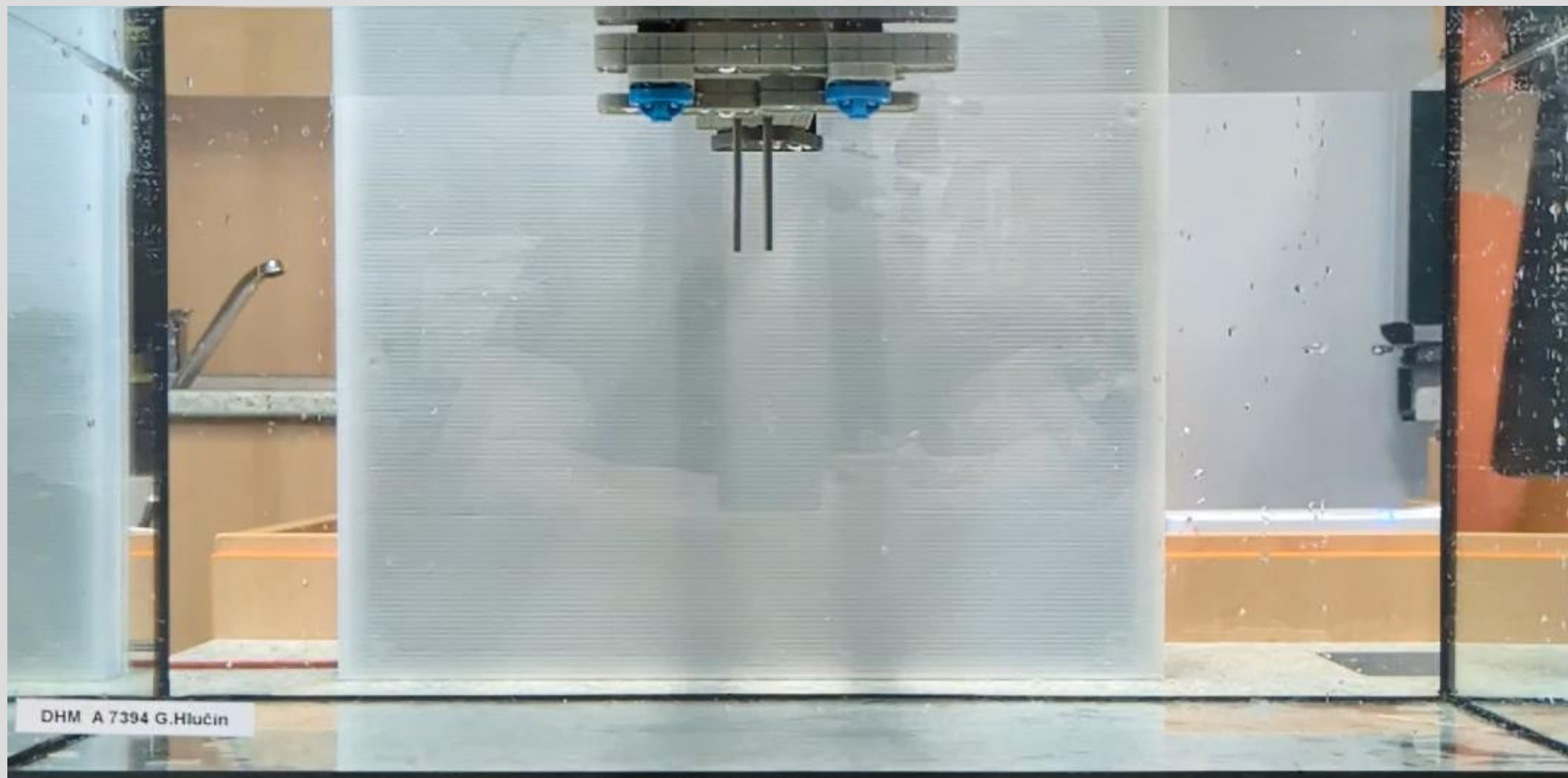


2D Phase Diagram (Thin Coins: $t/d < 0.1$)

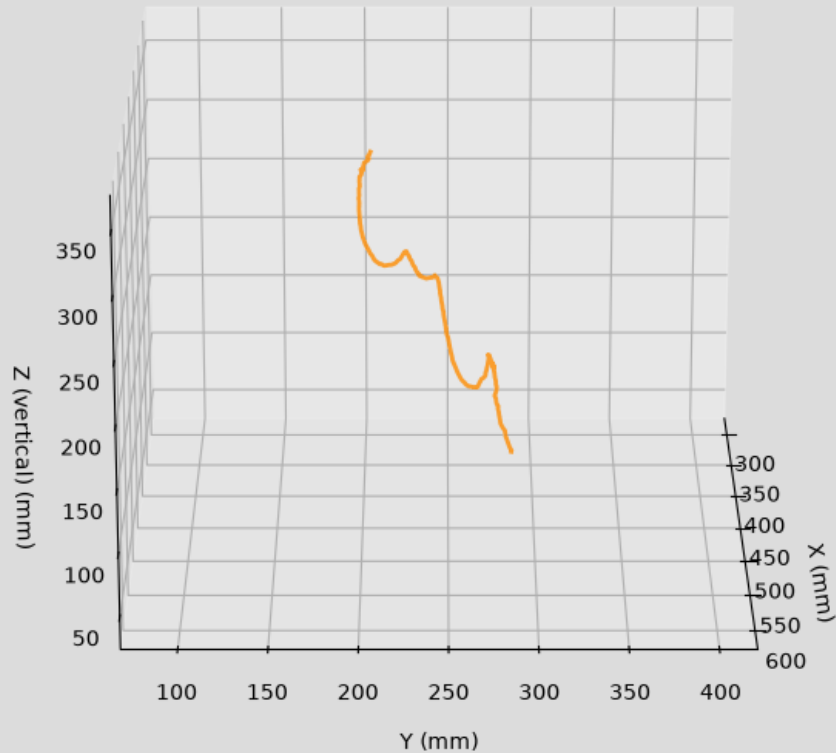
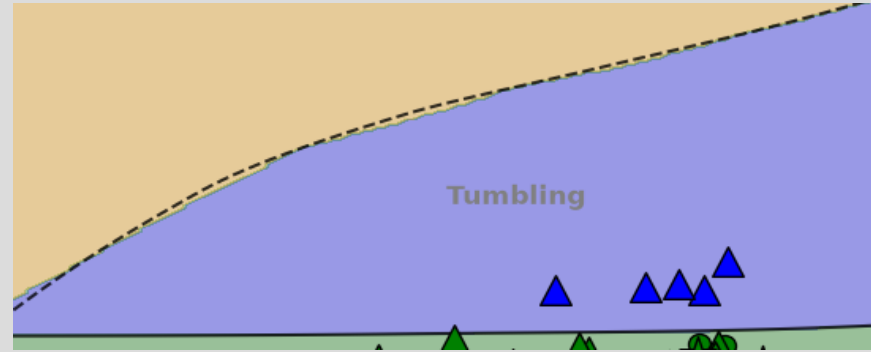


2D Phase Diagram (Thin Coins: $t/d < 0.1$)



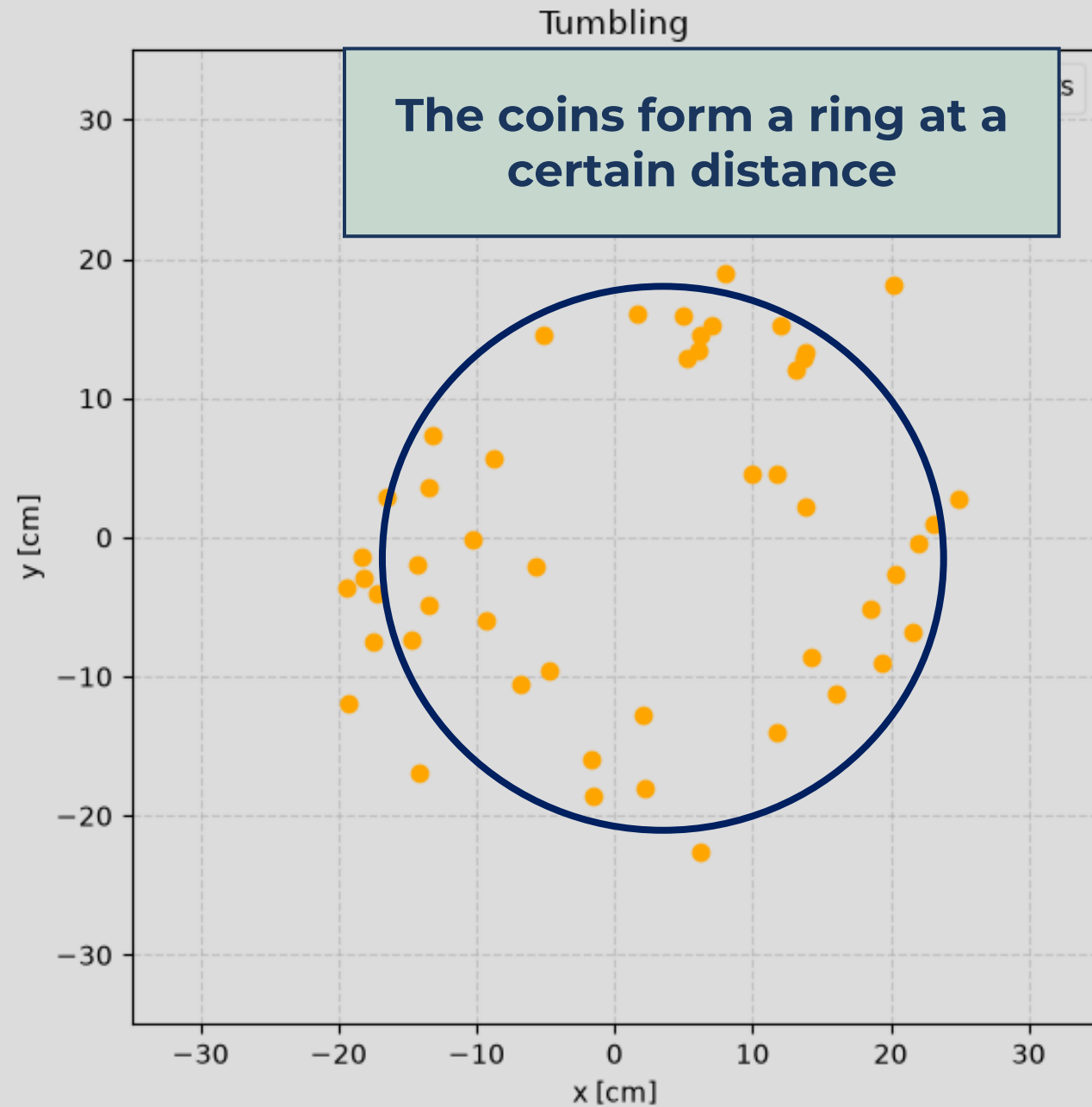


Tumbling

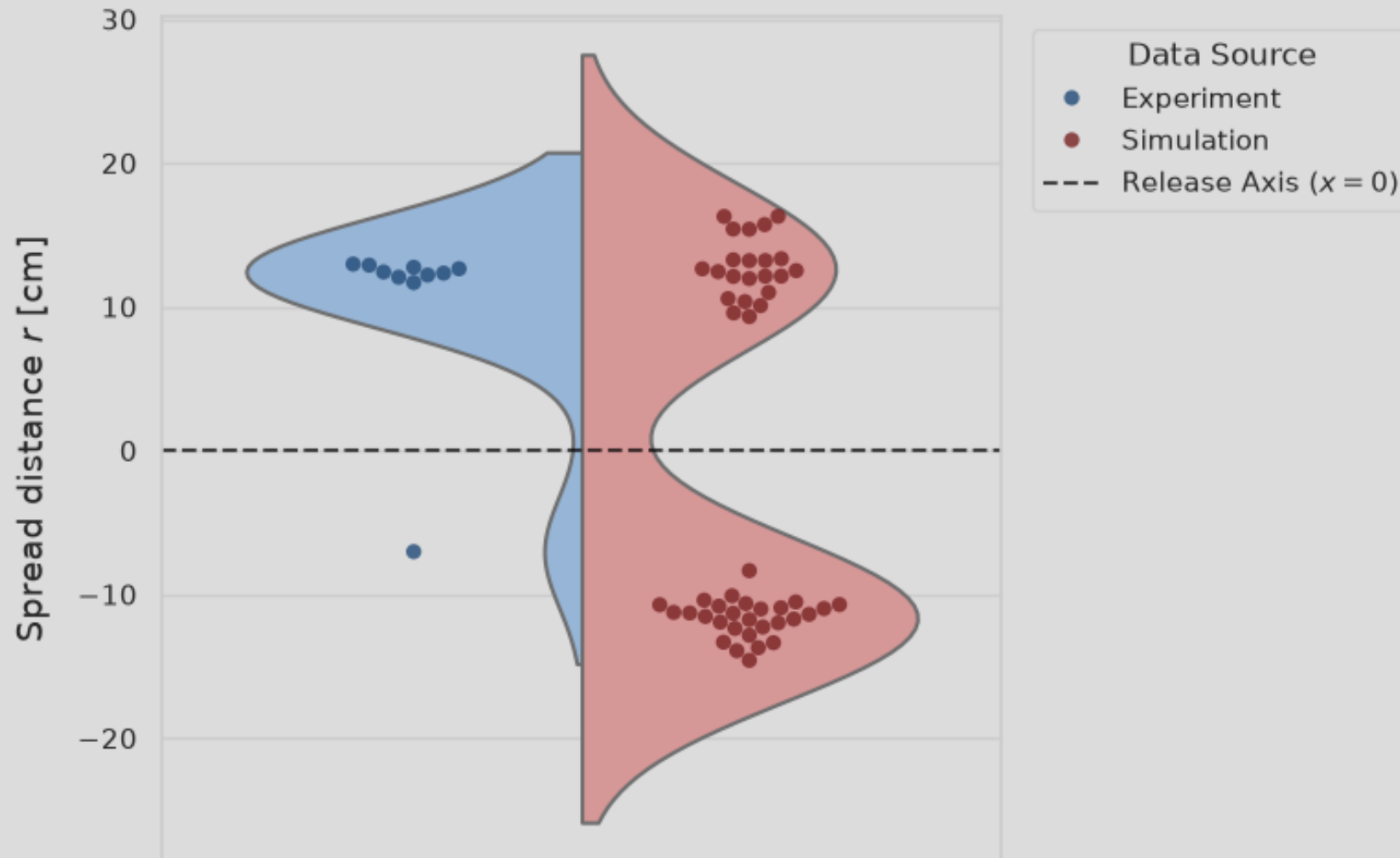


- Periodic motion
- **Autrotation**
- Higher sensitivity to initial conditions
- Lyapunov exponent:
 $\lambda = 3.2 \pm 0.3$

Dispersion



Dispersion prediction





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Conclusions

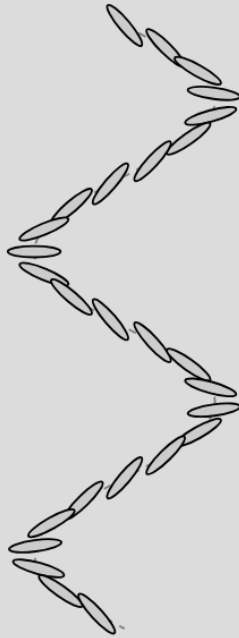
Qualitative predictions

Identified the **four modes** of motion 

(A) Steady



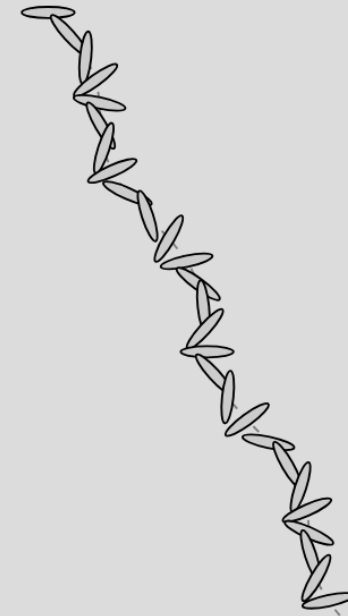
(B) Fluttering



(C) Chaotic



(D) Tumbling

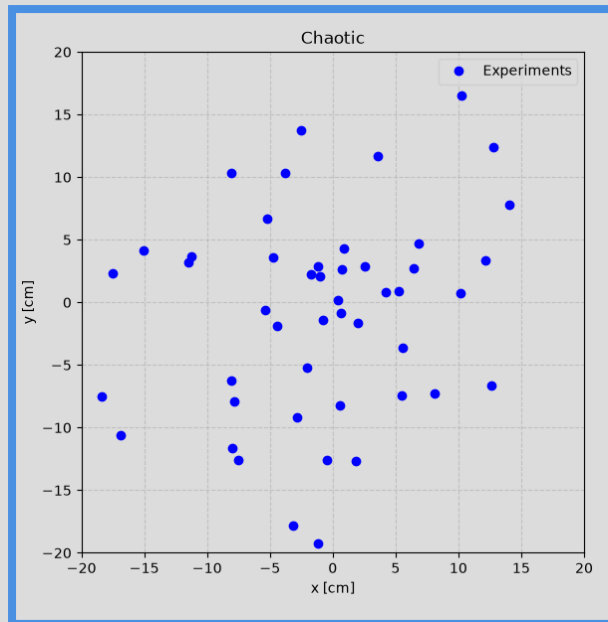


Conclusions

Qualitative predictions

Explained when **each mode happens** and mapped that to the **dimensionless numbers** ✓

Investigated what are **the characteristic properties** of each mode ✓



Reynolds number

$$Re = \frac{Ud}{\nu}$$

Dimensionless mass

$$m^* = \frac{\pi}{4} \left(\frac{\rho_s}{\rho} \right) \left(\frac{t}{d} \right)$$

Dimensionless moment of inertia

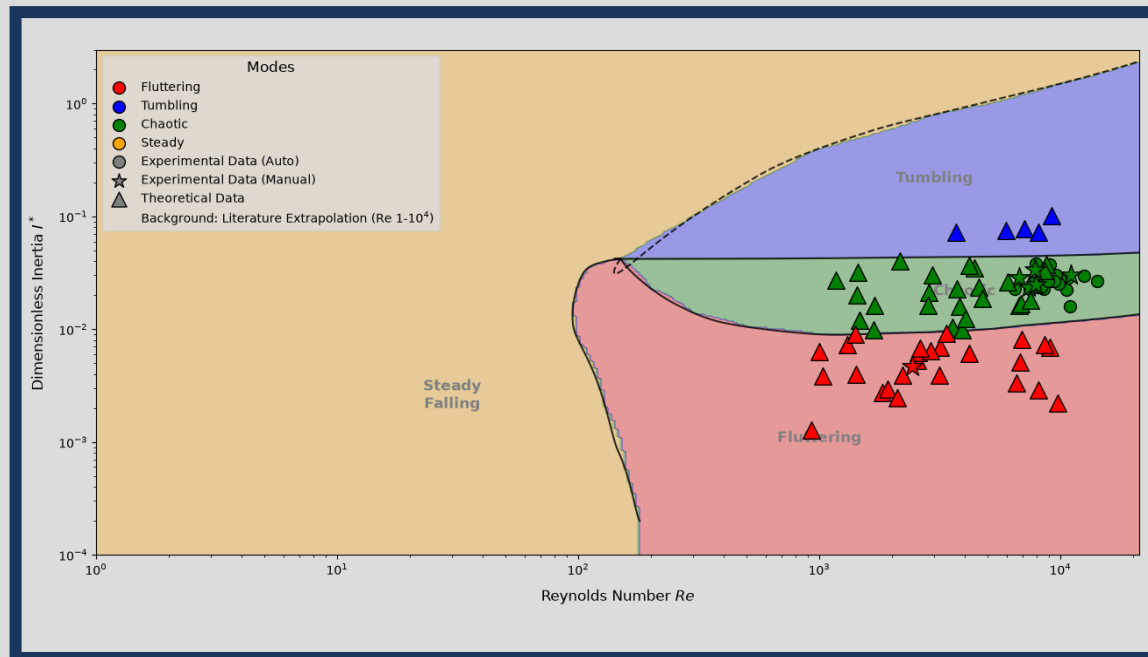
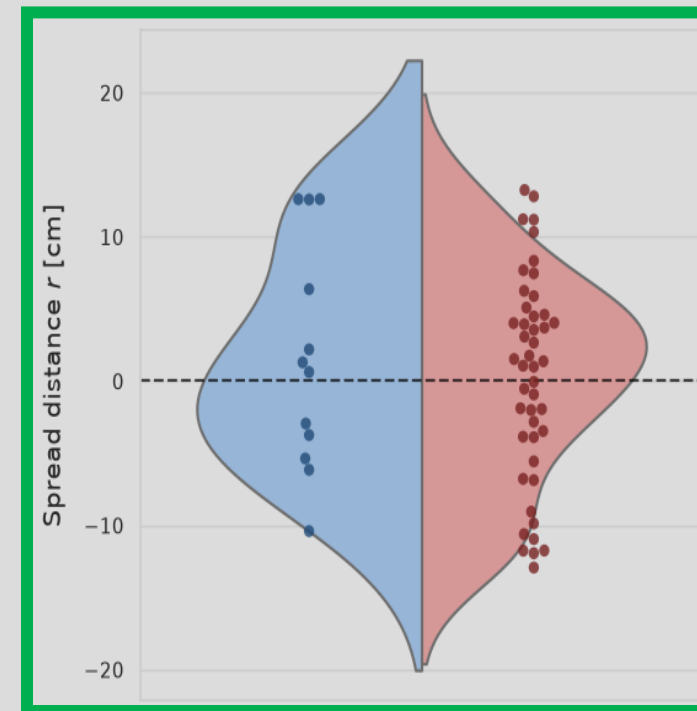
$$I^* = \frac{I}{\rho d^5}$$

Conclusions

Quantitative investigation

Develop a **CFD model** which matches our experiments ✓

Using **dimensionless numbers** we classified the modes ✓



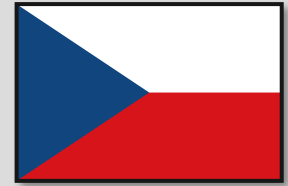
References

1. Heisinger, L., Newton, P., & Kanso, E. (2014). Coins falling in water. *J. Fluid Mech.*, 742, 243–253.
2. Kuznetsov, S. P. (2015). Plate Falling in a Fluid: Regular and Chaotic Dynamics of Finite-dimensional Models. *Regul. Chaotic Dyn.*, 20(3), 345–382.
3. Simonsen, A. S. (2015). Modelling of Plates in Free Fall: Development of a Predictive Trajectory Model by an Experimental and Numerical Approach. *Master's Thesis, Thermal Energy and Process Engineering*.
4. Field, S. B., Klaus, M., Moore, M. G., & Nori, F. (1997). Chaotic dynamics of falling disks. *Nature*, 388(6639), 252–254.



Thank you!

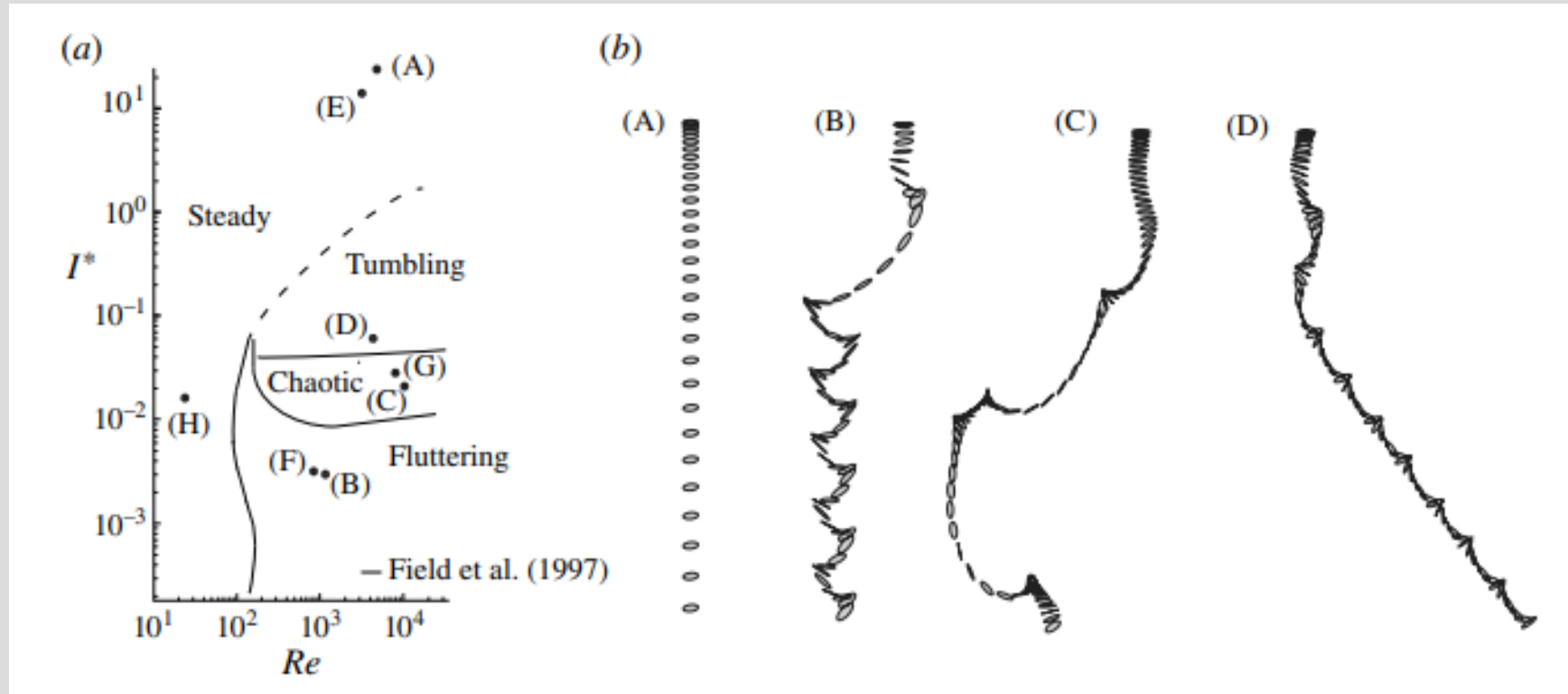
Kryštof Basista | Czech Republic



The motion of a coin falling to the bottom of a tank filled with liquid can be remarkably similar to the fluttering and tumbling of a falling autumn leaf. Investigate how the motion of the coin depends on relevant parameters.

Open  FOAM

Mode conditions



Dimensionless
moment of inertia

$$I^* = \frac{I}{\rho_w d^5}$$

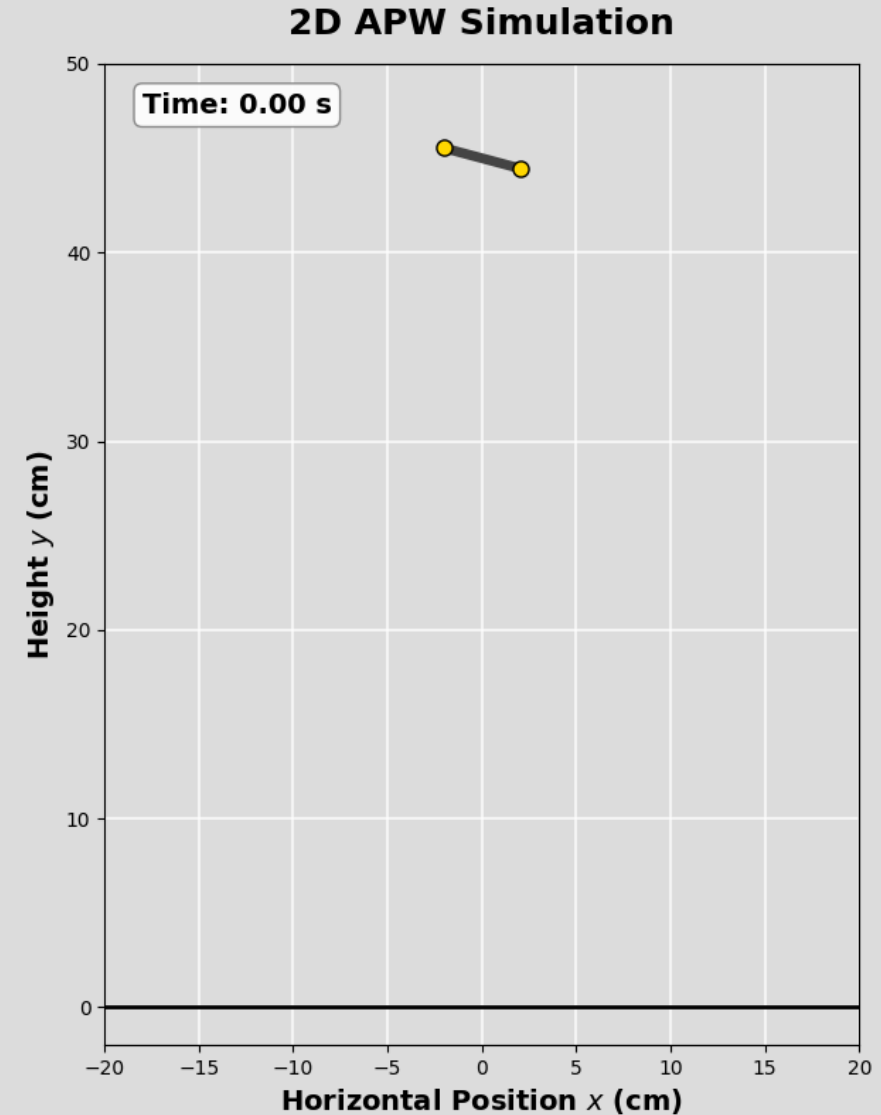
Reynold's number

$$Re = \frac{hd}{tv}$$

Luke Heisinger, Paul Newton and Eva Kanso. (2014)

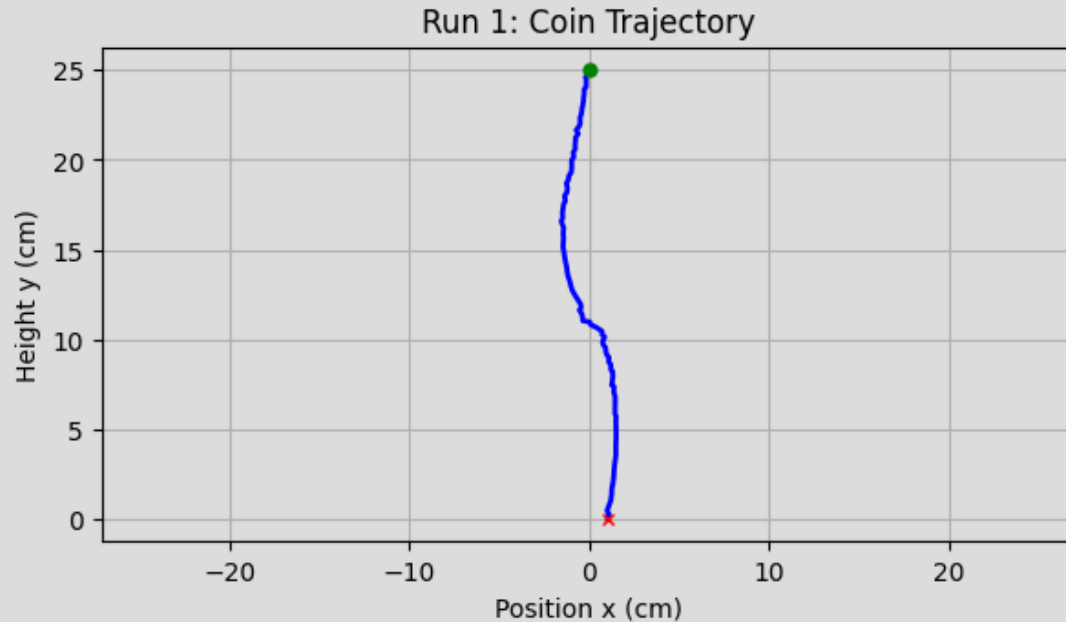
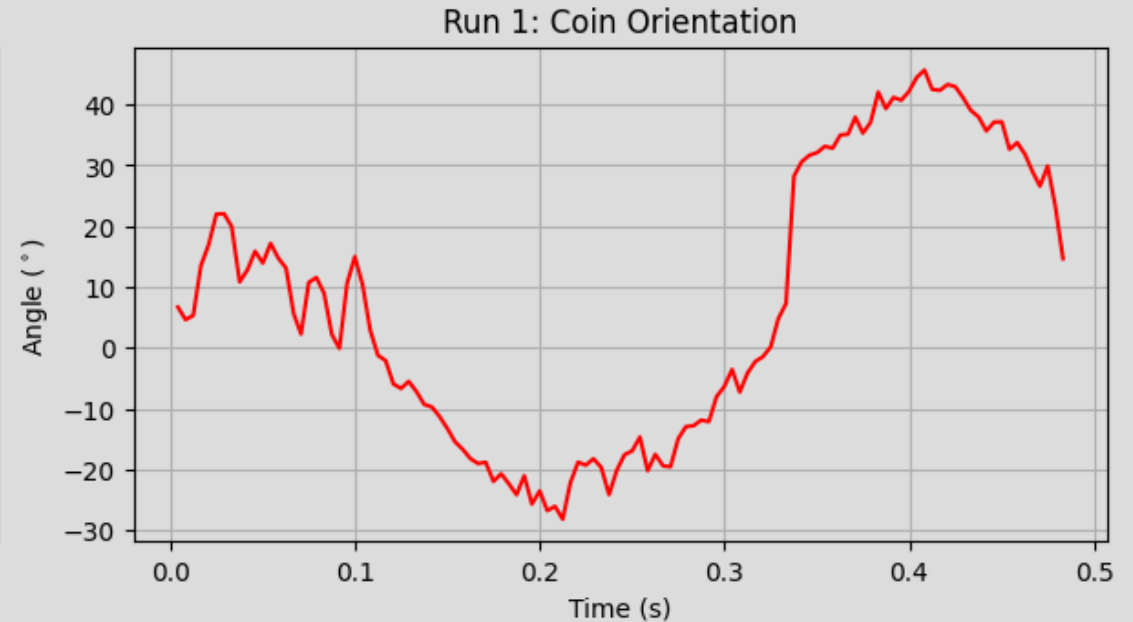
Dynamic model

- Added mass
- Circulation
- Viscous drag
- Bouyancy
- Forces approach



Appendix – high effective mass

Coin Analysis (Detected 5 Runs)



Dimensionless numbers – π Buckingham theorem

- d Coin diameter
- U Average coin vertical velocity
- ν Kinematic viscosity of water
- t Coin thickness
- ρ_s Coin material density
- ρ Water density
- g gravitational acceleration

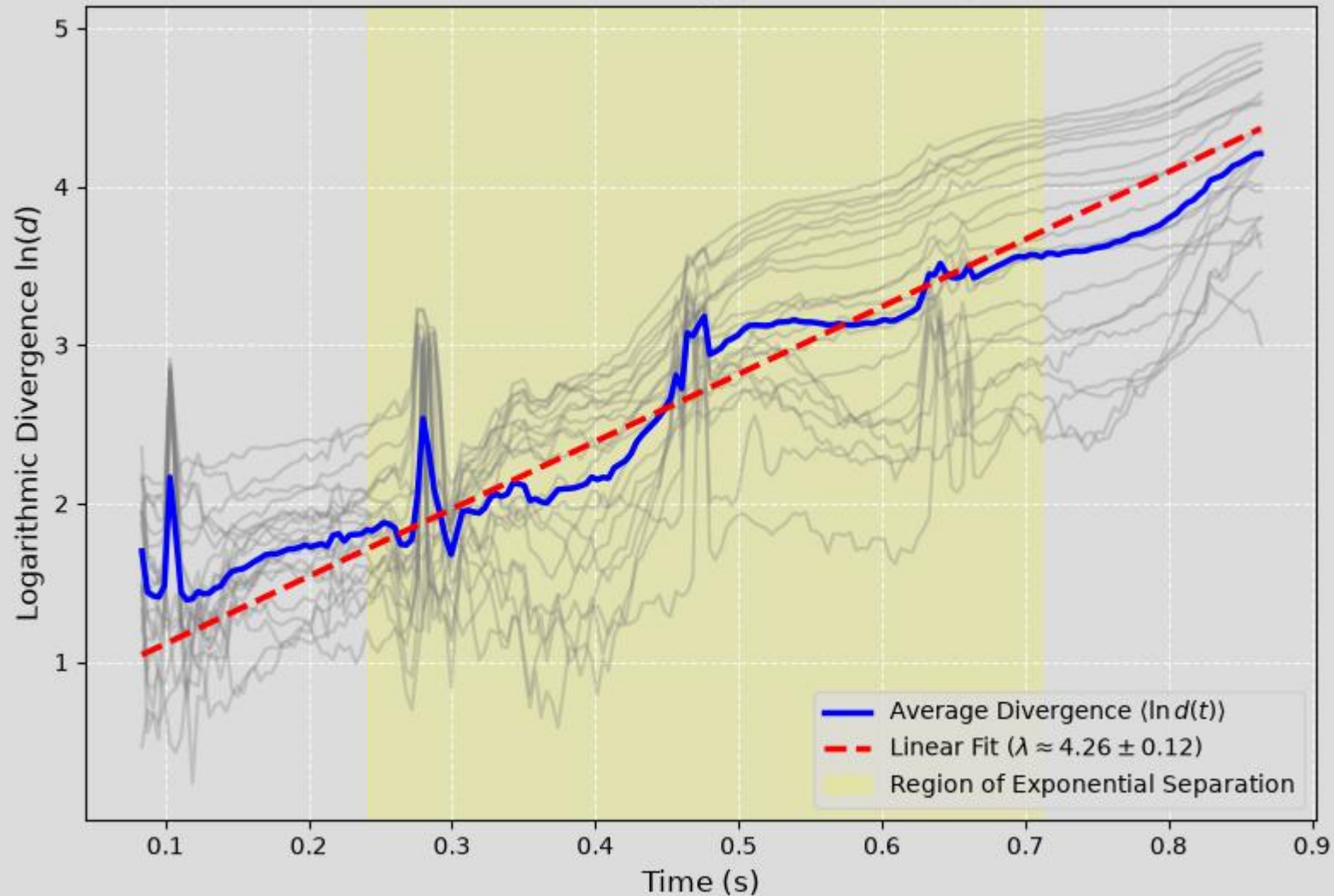
7 independent variables

3 independent physical dimensions

$$p = n - k$$

- p Number of dimensionless numbers
- n independent variables
- k independent physical dimensions

Lyapunov exponent estimation



Appendix - 2D APW model

$$\beta = \frac{b}{a}$$

Added inertia

Centrifugal

Circulation

Drag

$$(I^* + \beta^2)\dot{u} = (I^* + 1)v\dot{\theta} - \bar{\Gamma}v - \sin\theta - Ku,$$

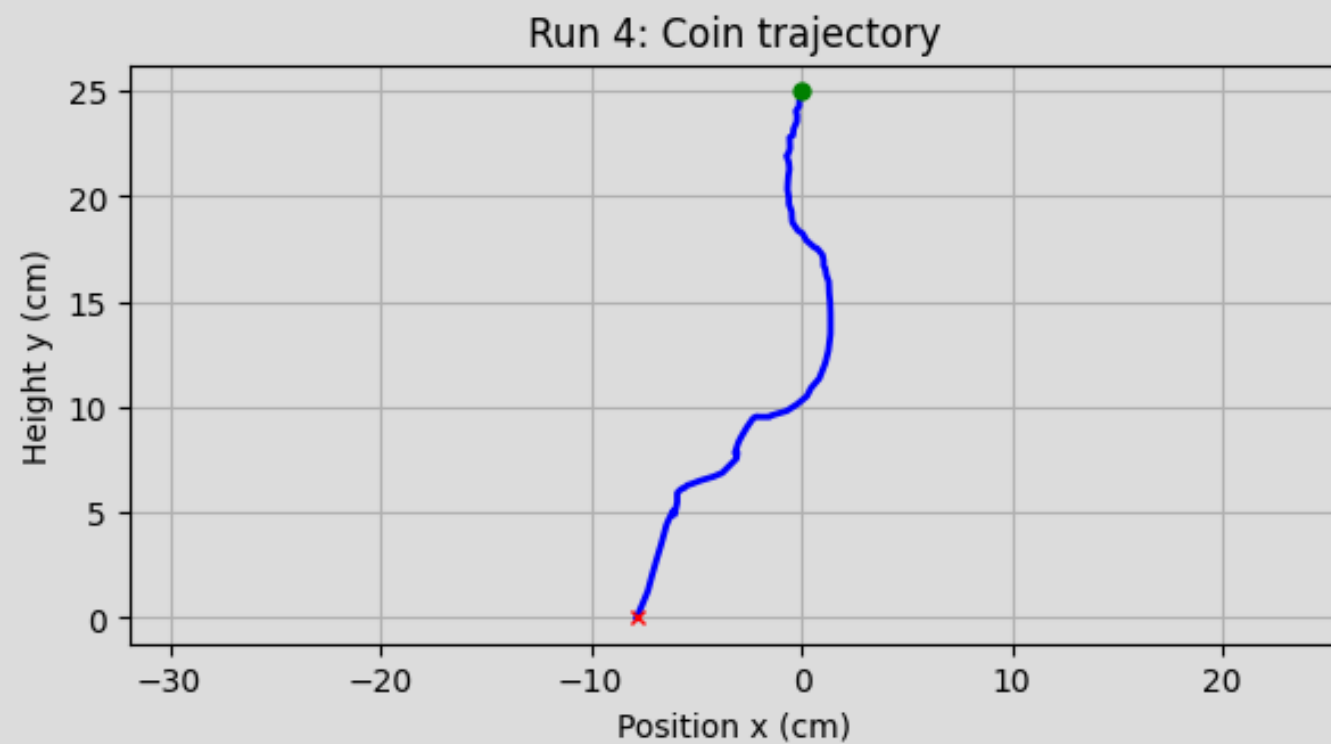
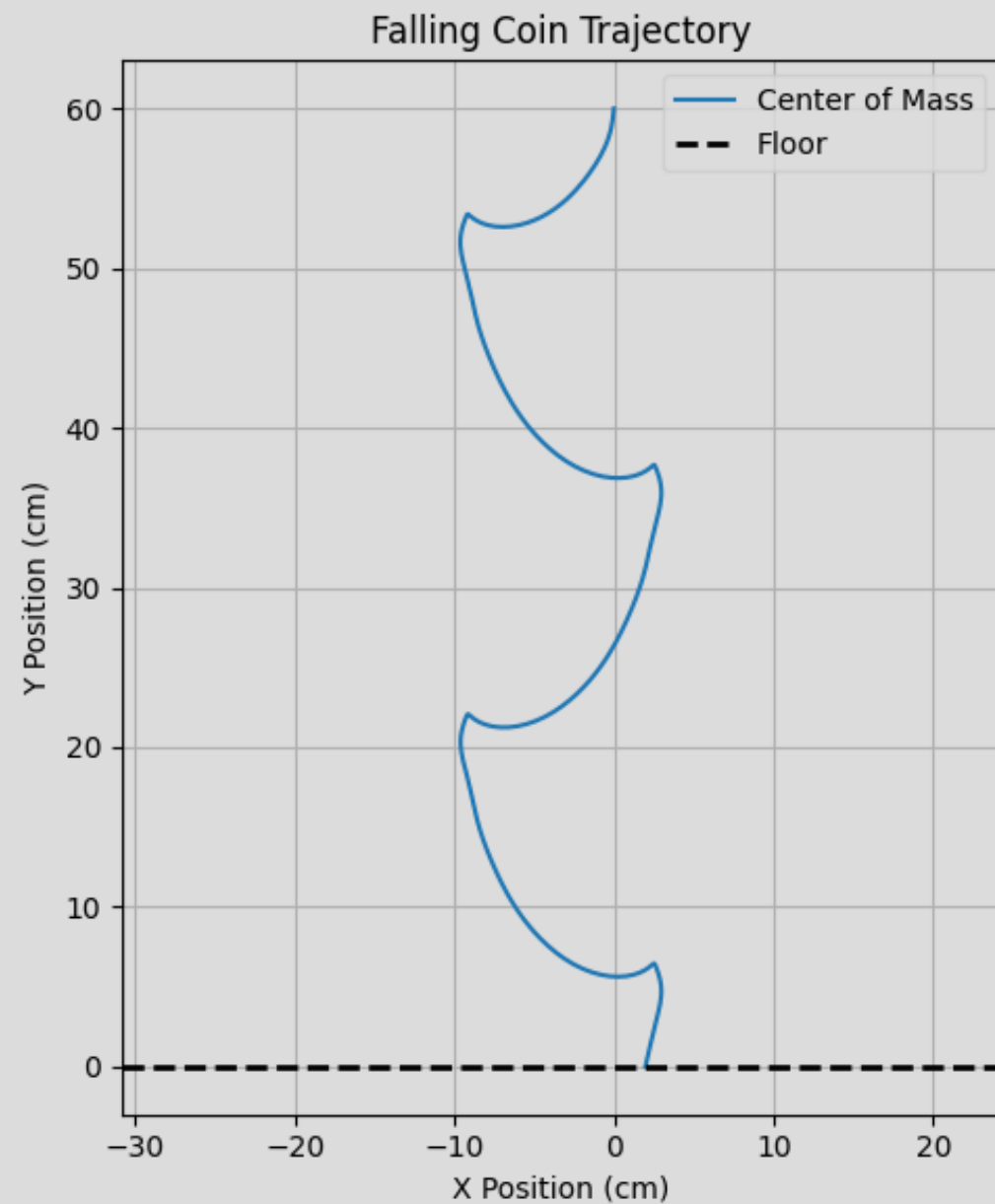
$$(I^* + 1)\dot{v} = -(I^* + \beta^2)u\dot{\theta} + \bar{\Gamma}u - \cos\theta - Kv,$$

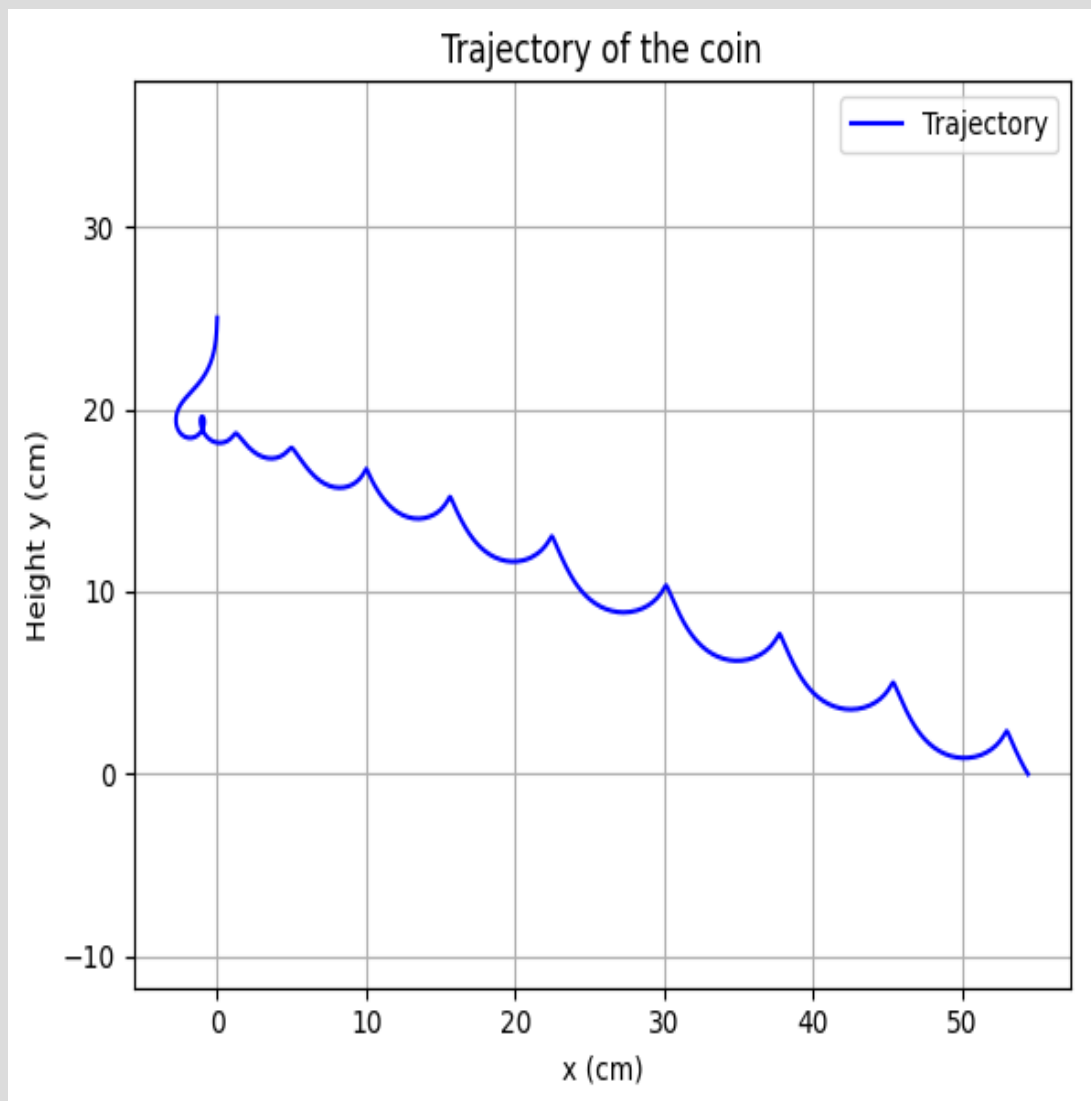
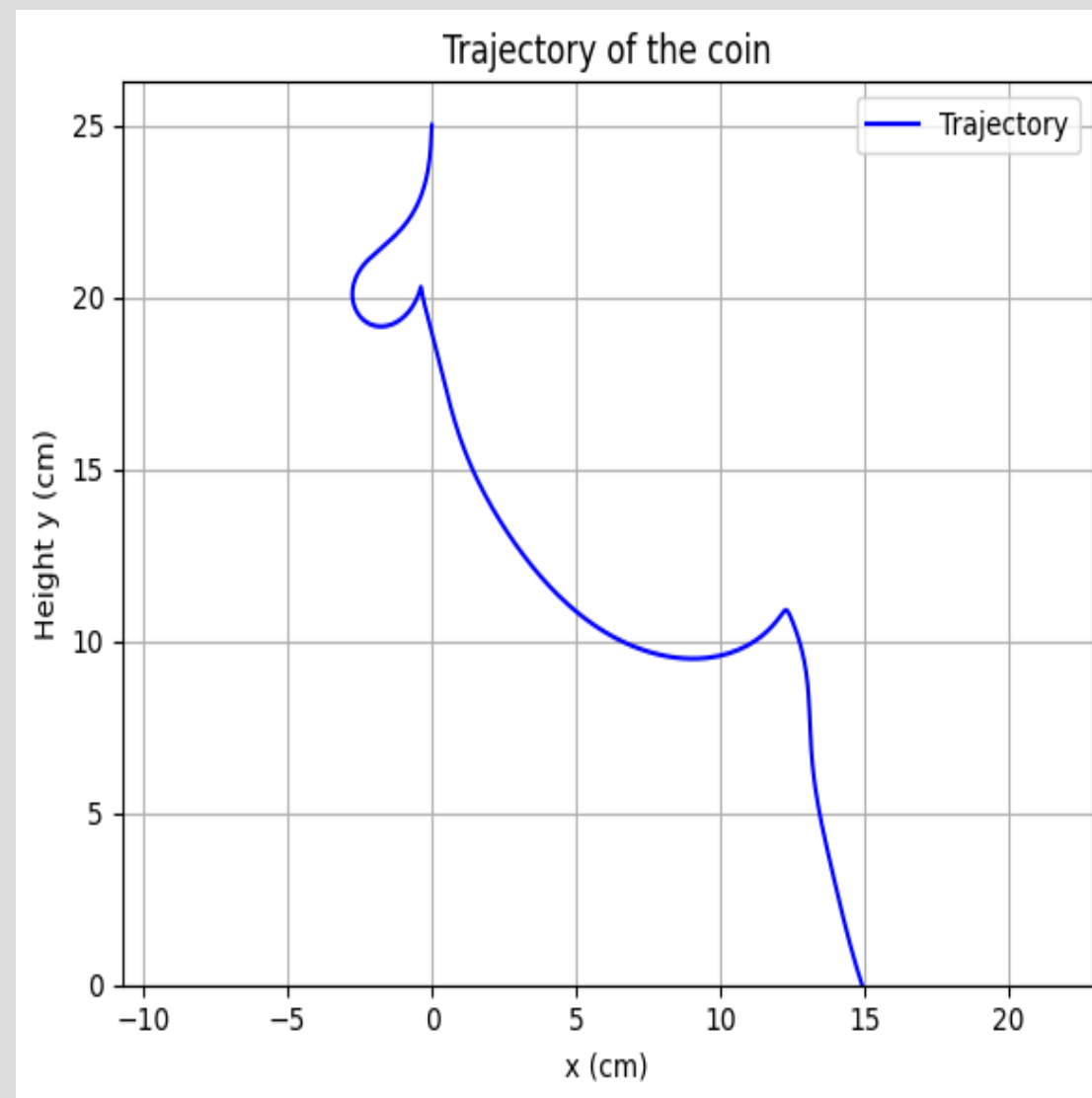
$$\left(\frac{1}{4}(1 + \beta^2)I^* + \frac{1}{8}(1 - \beta^2)^2\right)\ddot{\theta} = (\beta^2 - 1)uv - K_\theta\dot{\theta}.$$

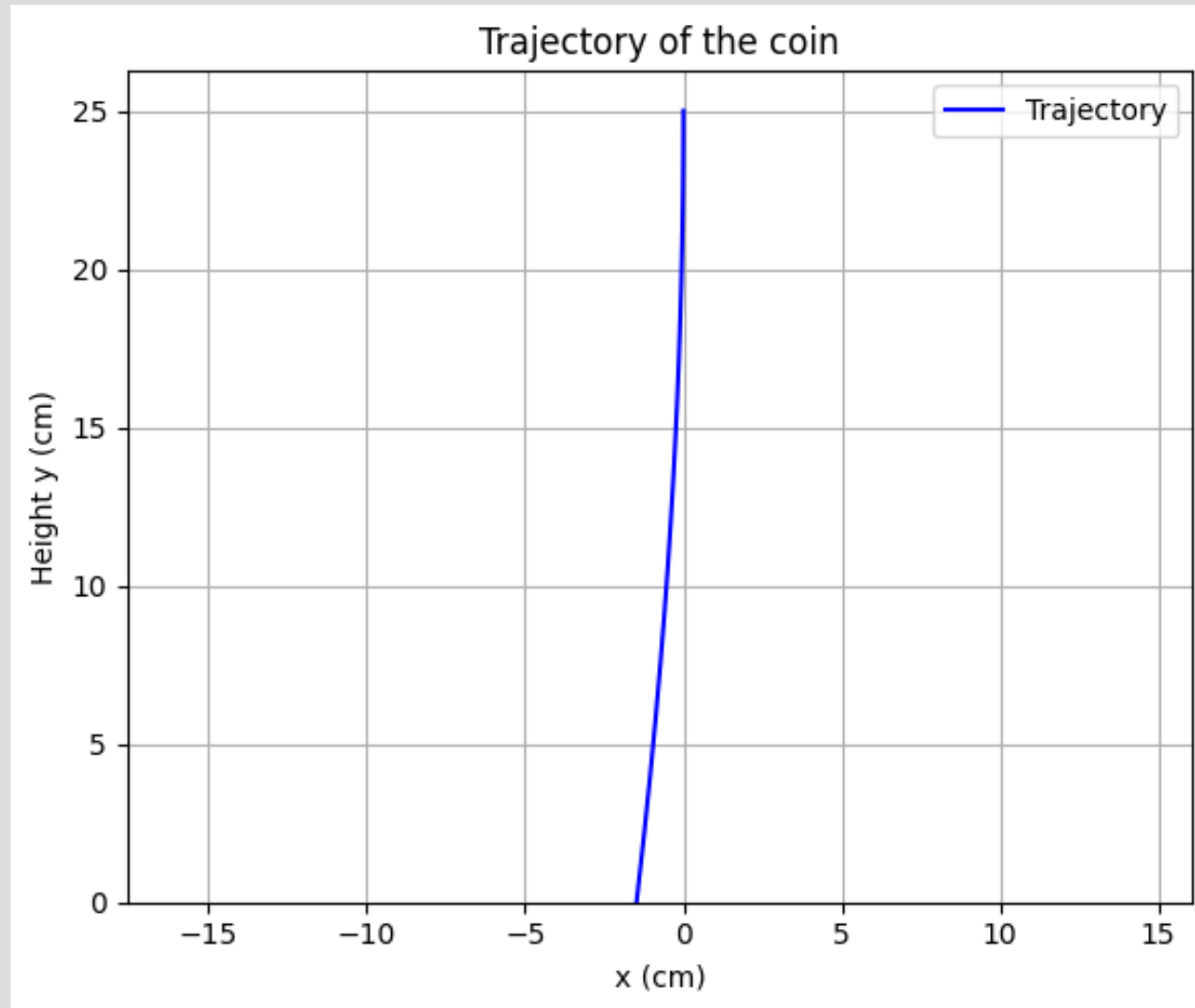
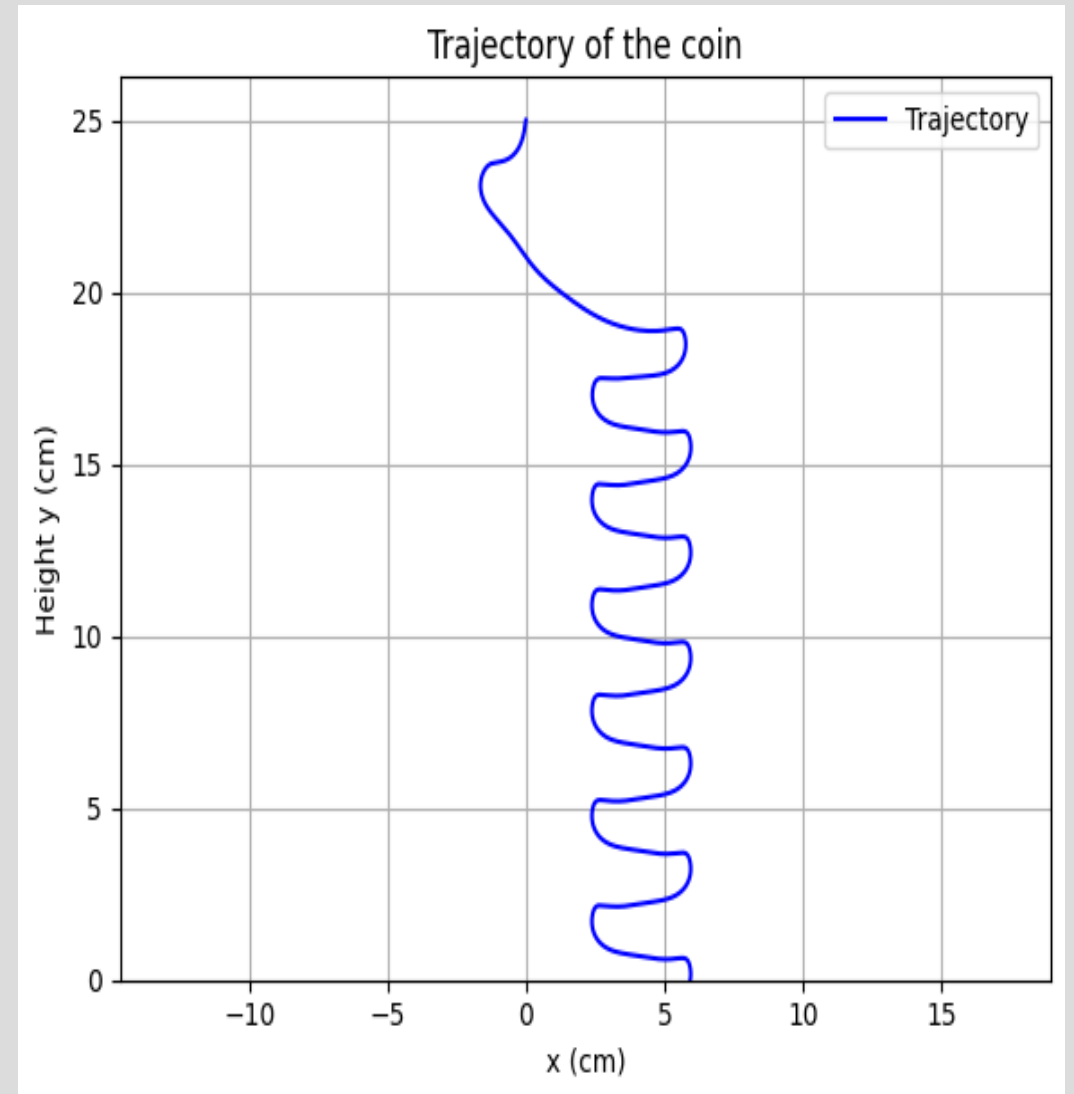
$$\bar{\Gamma} = \frac{2}{\pi} \left(-C_T \frac{uv}{\sqrt{u^2 + v^2}} + C_R\dot{\theta} \right), \quad K = \frac{1}{\pi} \left(C_A - C_B \frac{u^2 - v^2}{u^2 + v^2} \right) \sqrt{u^2 + v^2},$$

$$K_\theta = \mu'_1 + \mu_2|\dot{\theta}|, \quad \mu'_1 = \mu_1\sqrt{gI^*(1 - \rho)}.$$

Kuznetsov, S. P. (2015). Plate Falling in a Fluid: Regular and Chaotic Dynamics of Finite-dimensional Models. *Regul. Chaotic Dyn.*, 20(3), 345–382.



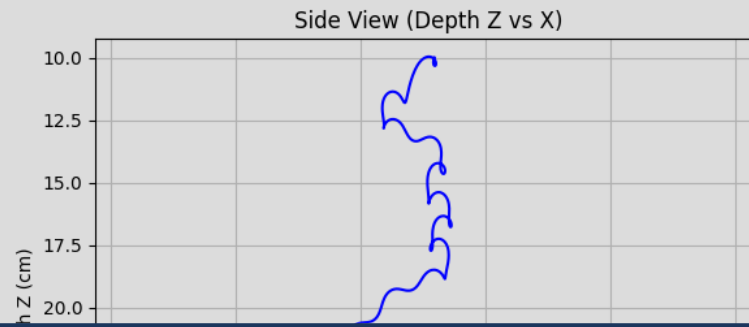
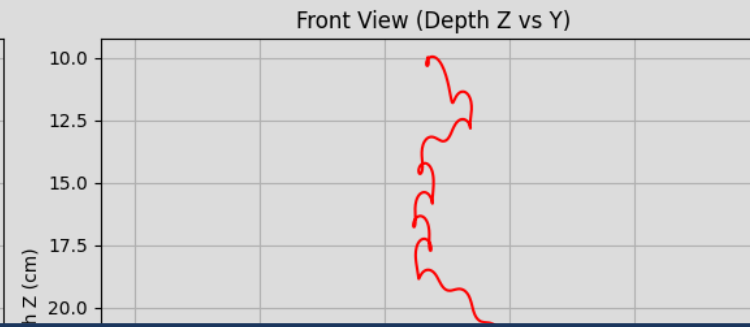
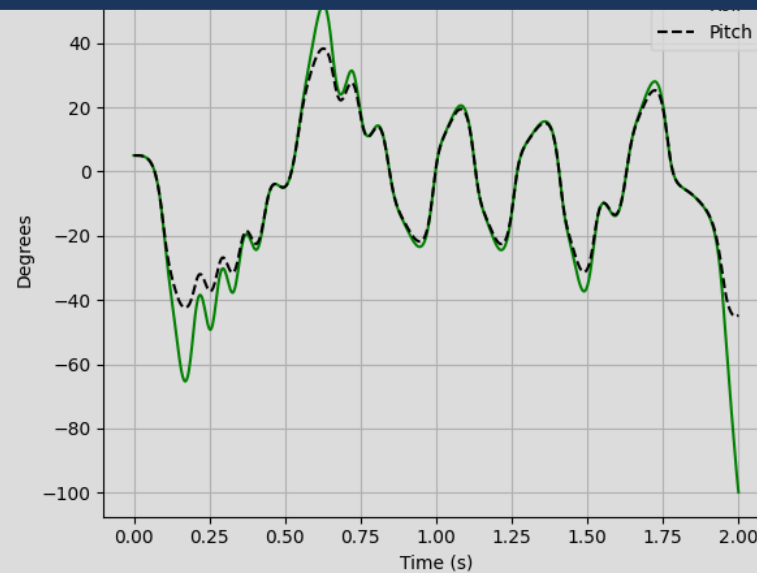
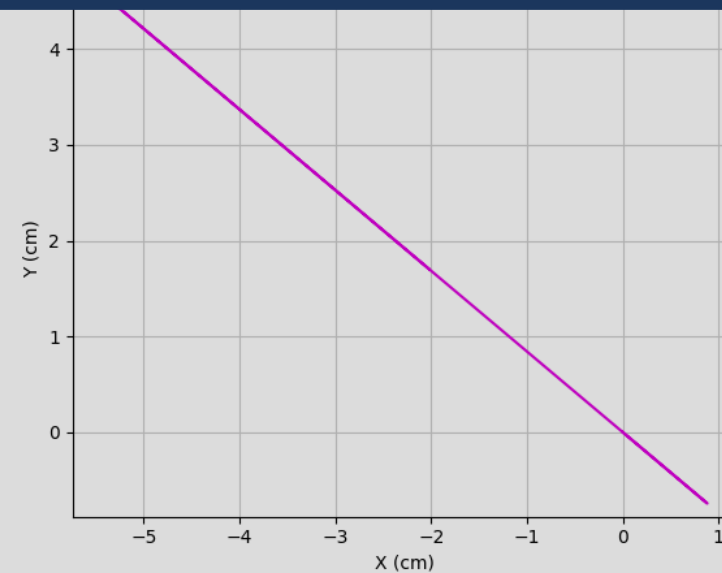




3D AI model



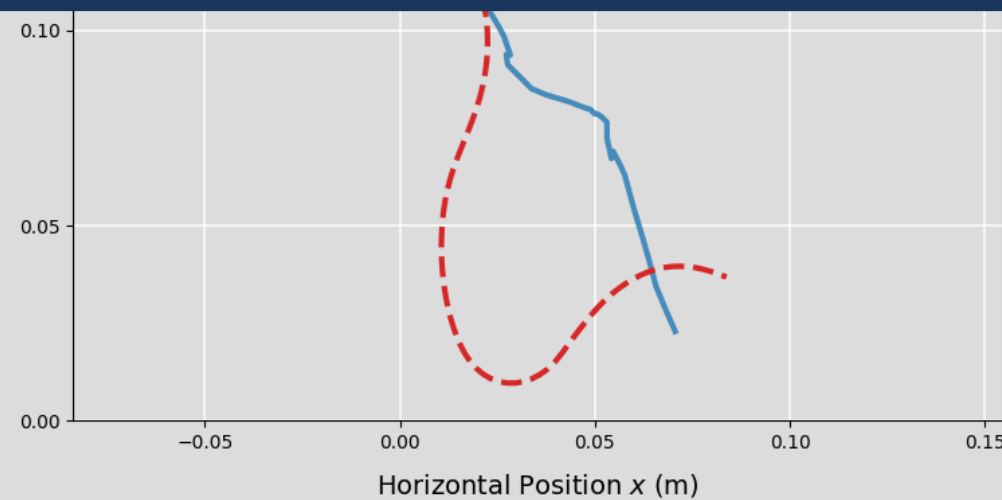
Unrealistic simulation





Less vortices = less inertia

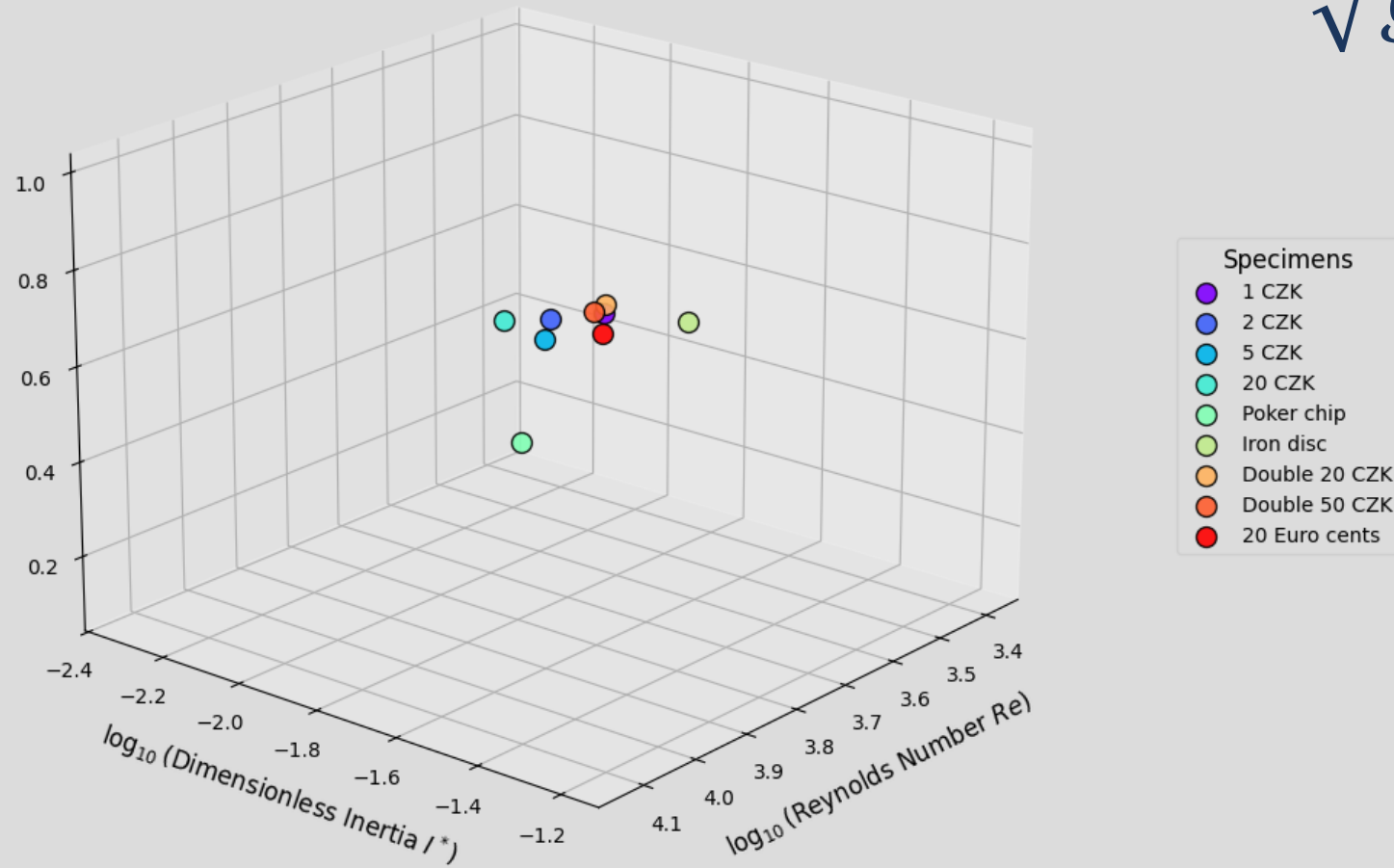
present



3D Phase Diagram: Influence of Froude Number

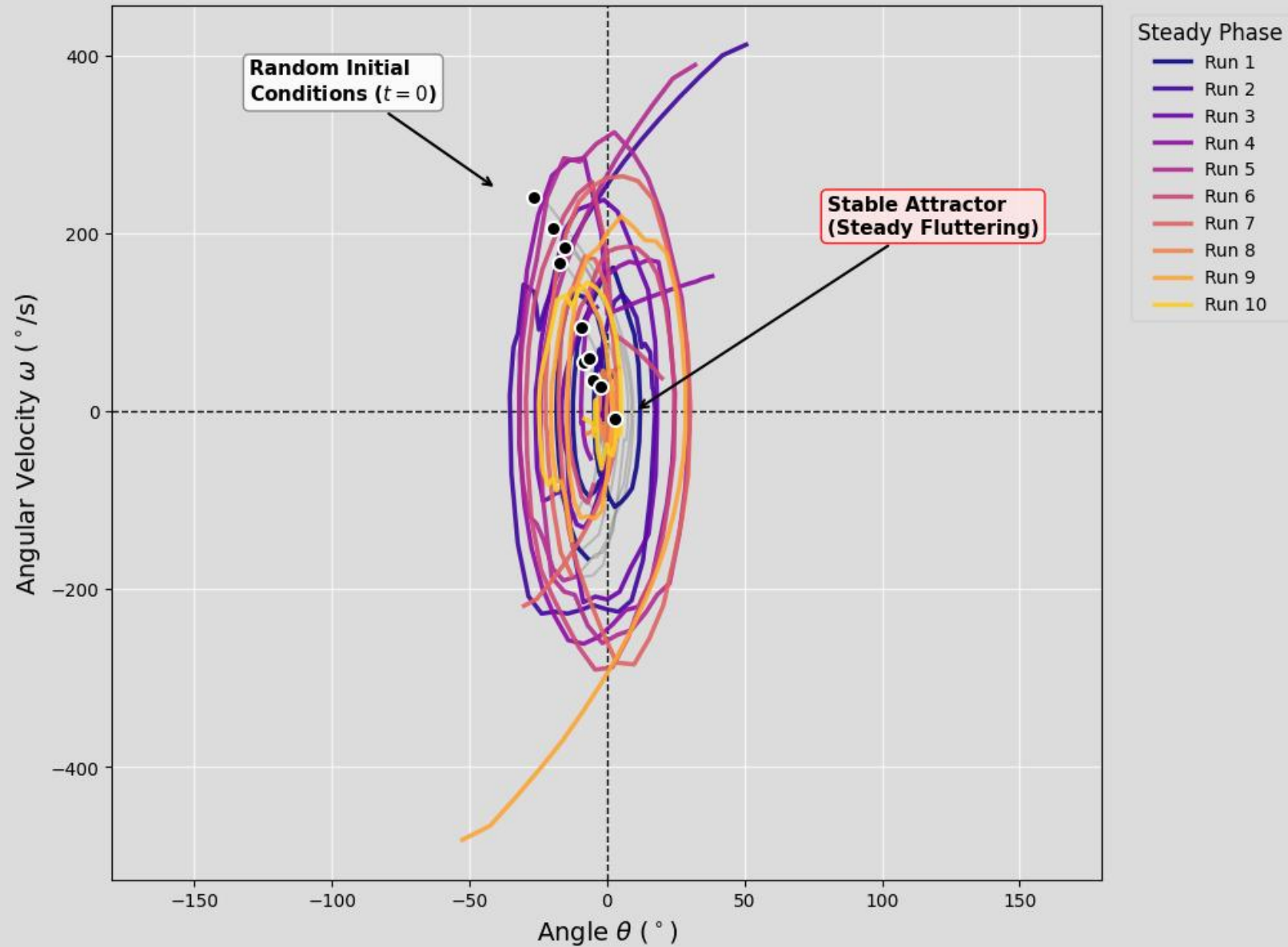
$$Fr = \frac{U}{\sqrt{gd}}$$

Froude number Fr



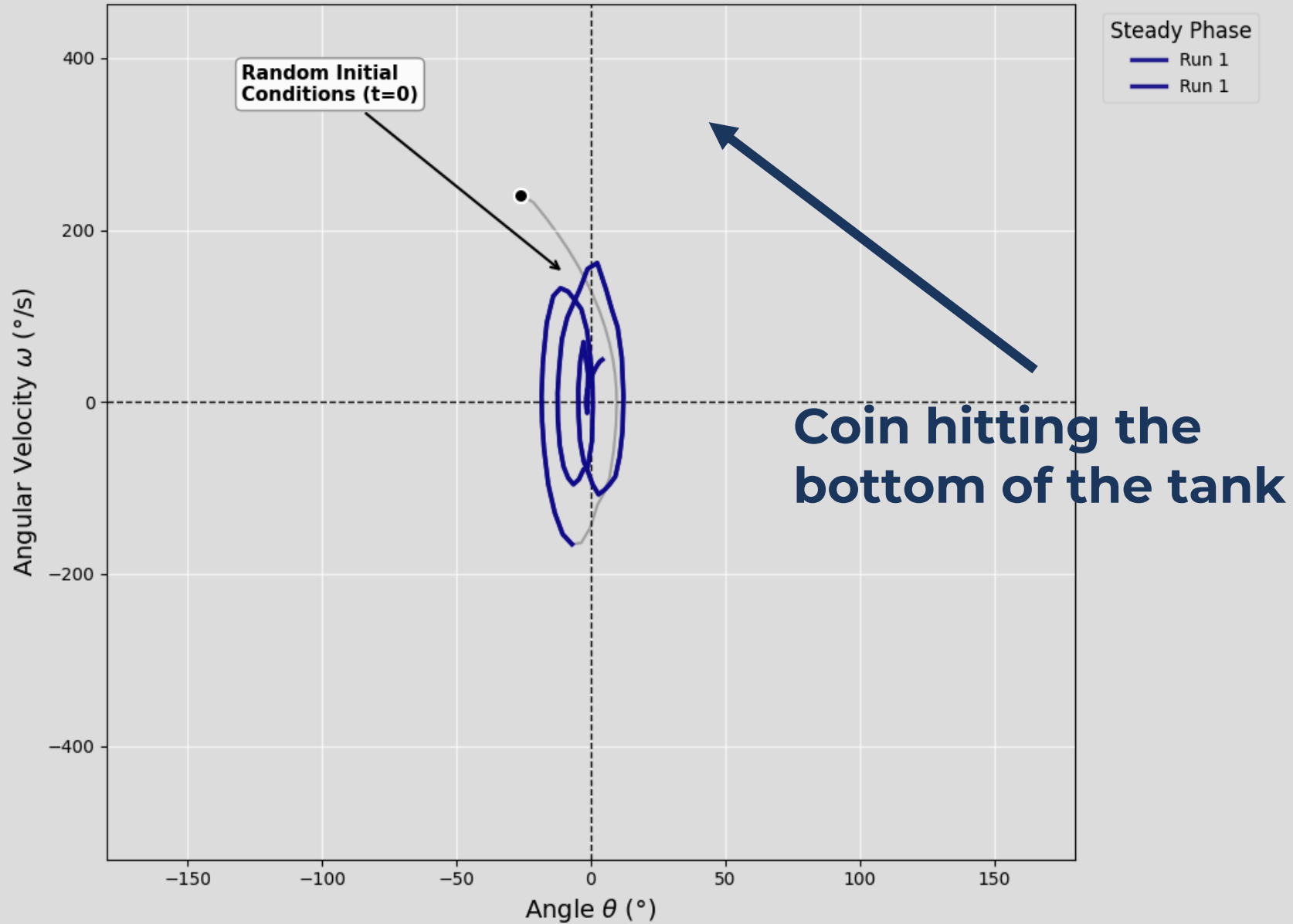
Basin of Attraction: Phase Space Fluttering

Initial chaotic release (grey) converges to a stable fluttering limit cycle (colored)



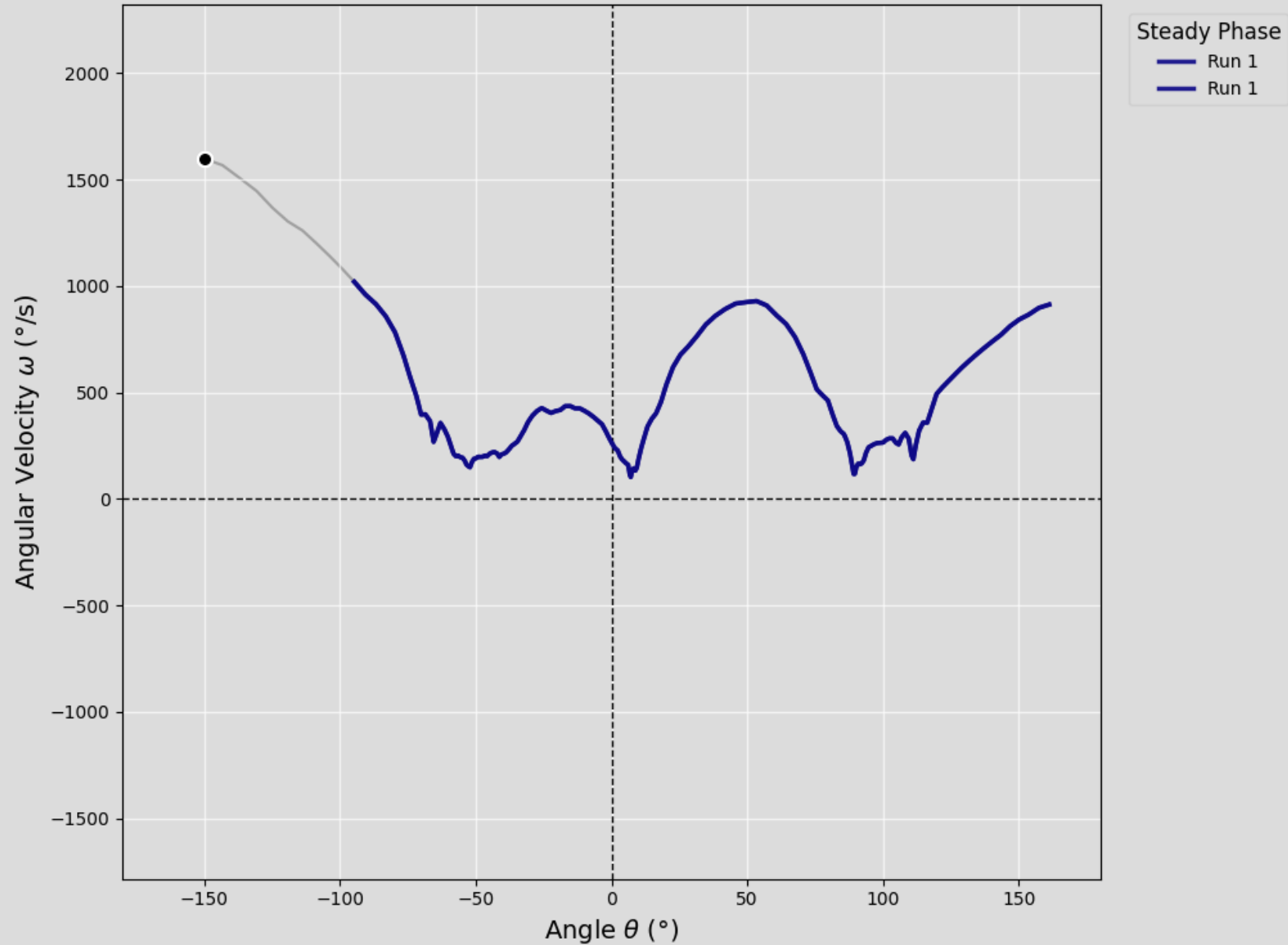
Phase Space Fluttering

Random initial conditions lead to the same motion regime

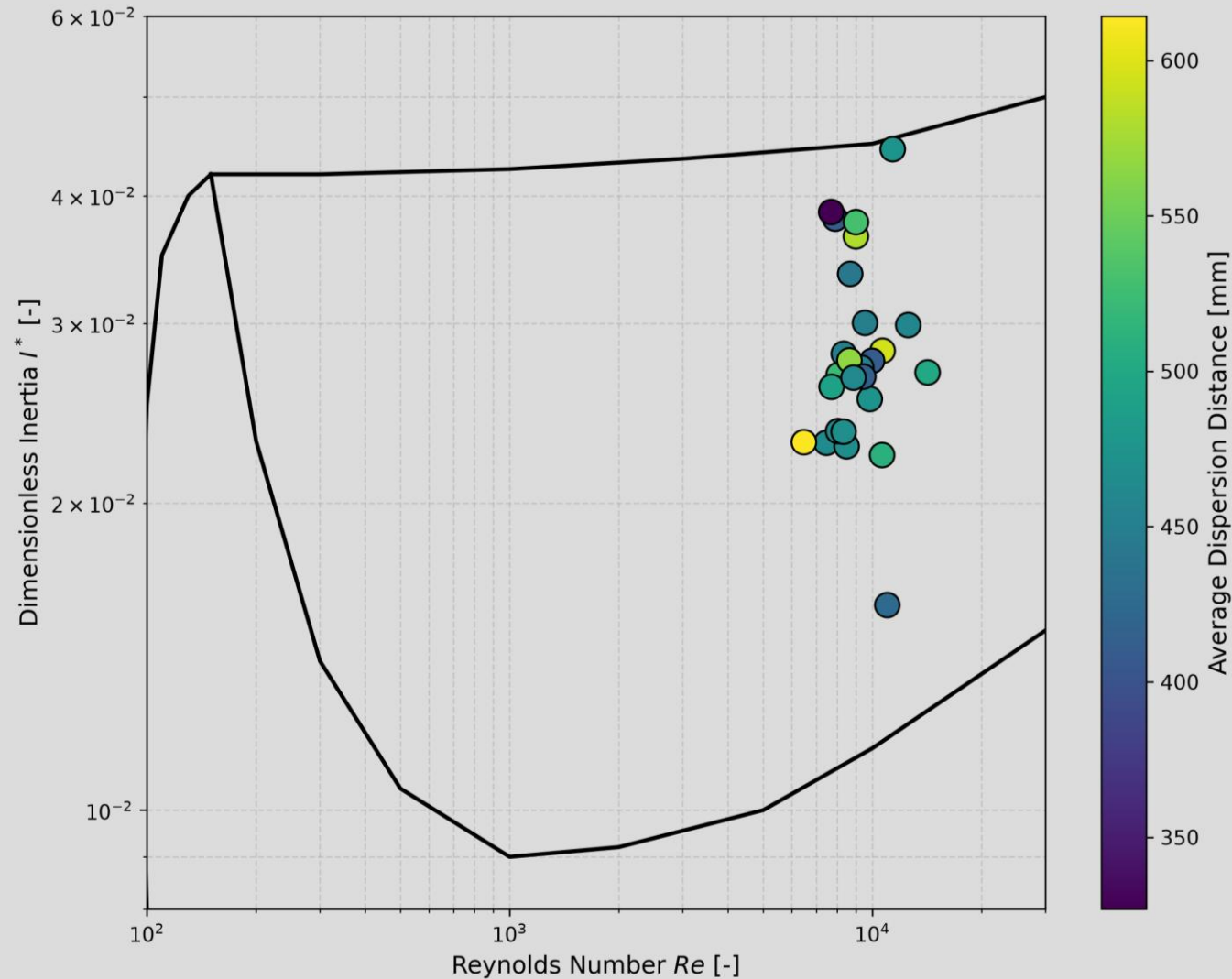


Phase Space Tumbling

Random initial conditions lead to the same motion regime

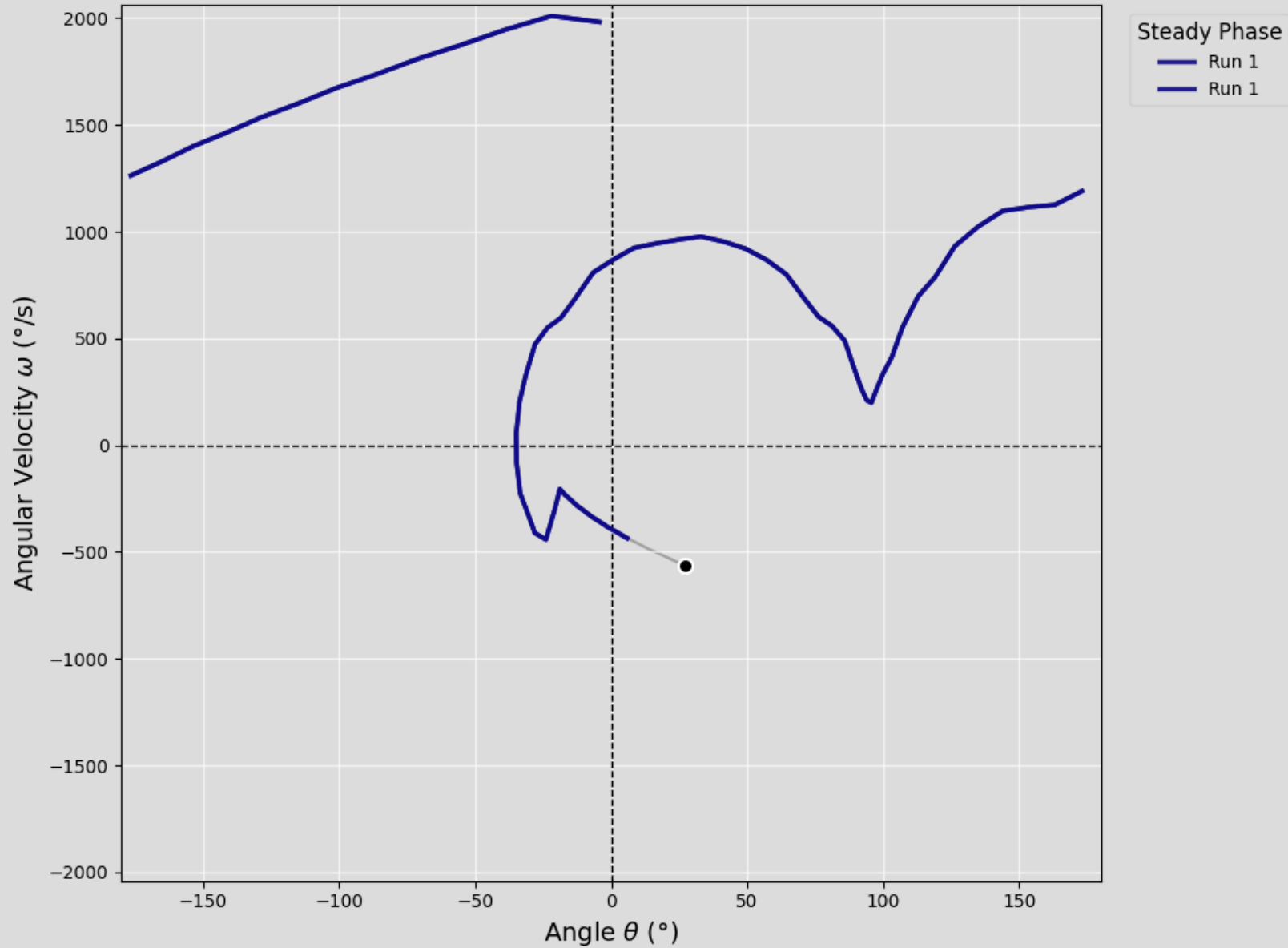


Appendix - Spread map

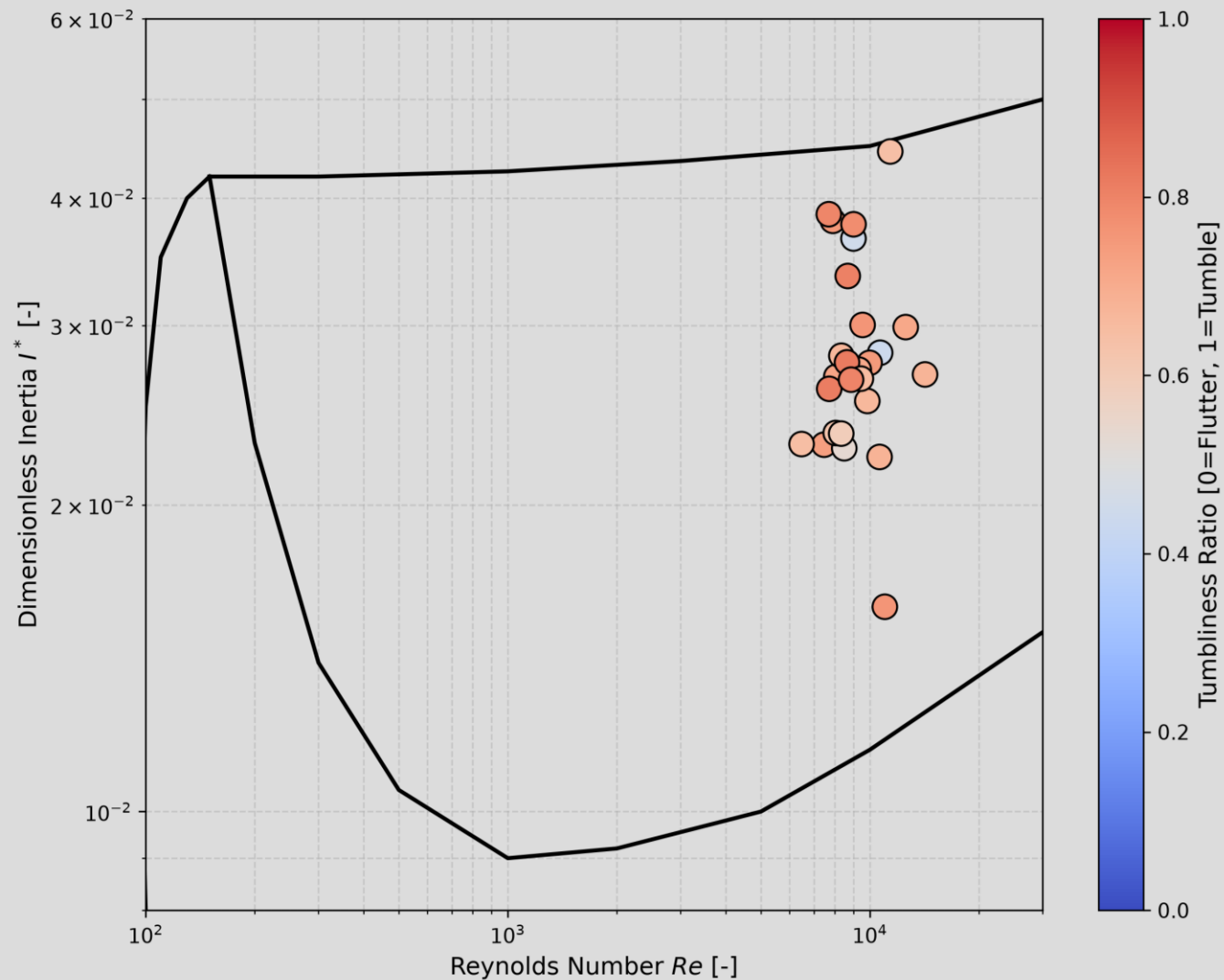


Phase Space Chaotic

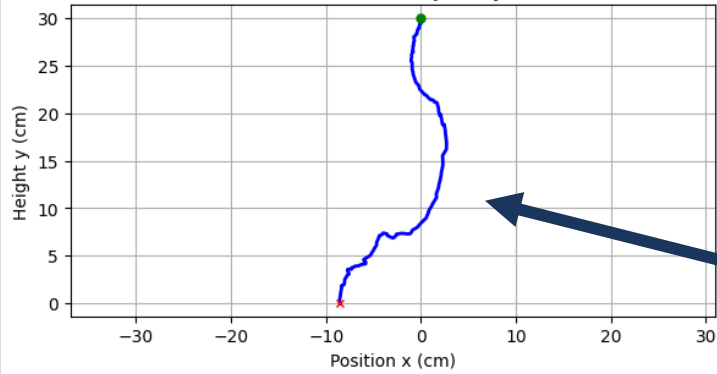
Random initial conditions lead to the same motion regime



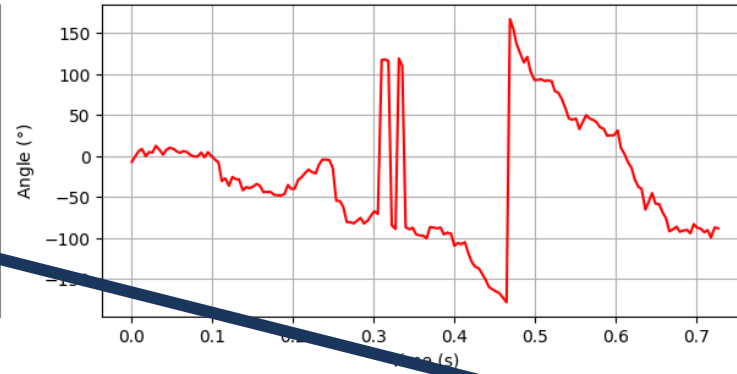
How chaotic?



Run 3: Coin Trajectory

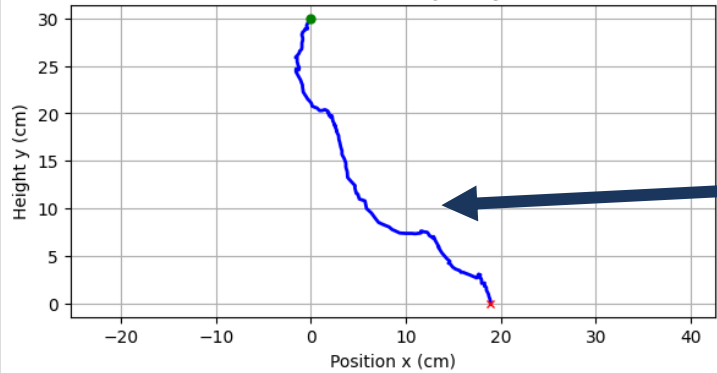


Coin Orientation

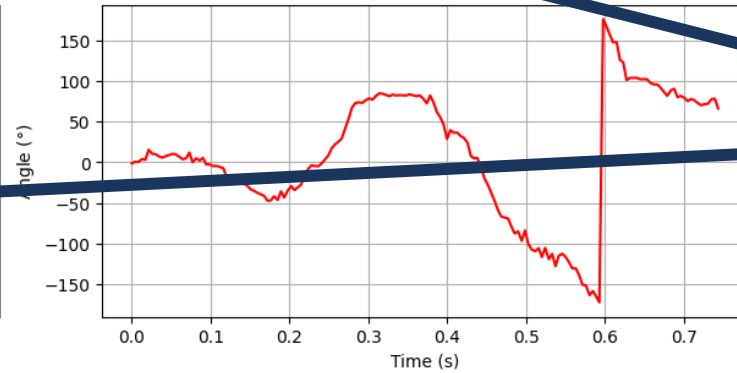


$d = 2,15 \text{ cm}$
 $t = 1,85 \text{ mm}$
 $m = 3,7 \text{ g}$

Run 4: Coin Trajectory

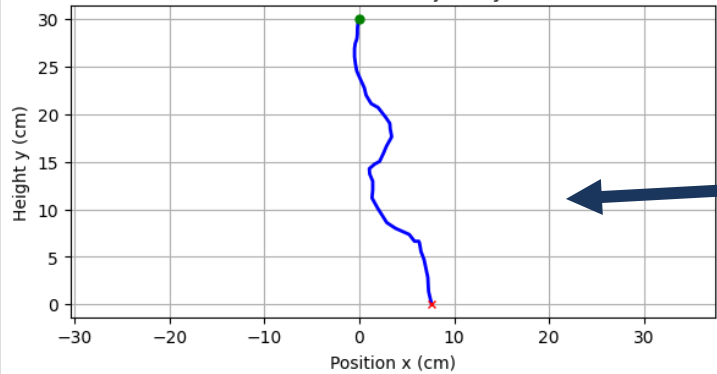


Coin Orientation

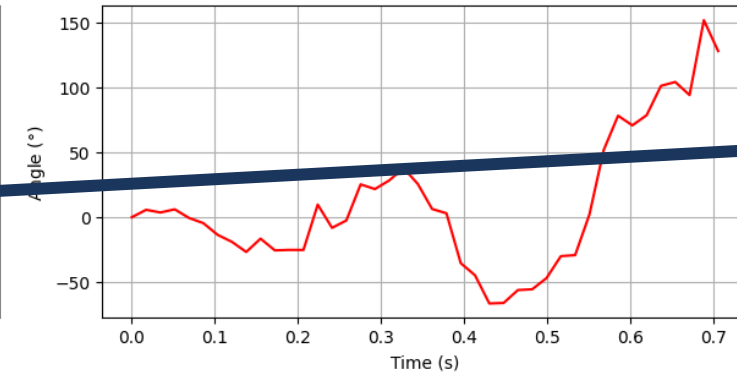


Tumbling

Run 5: Coin Trajectory



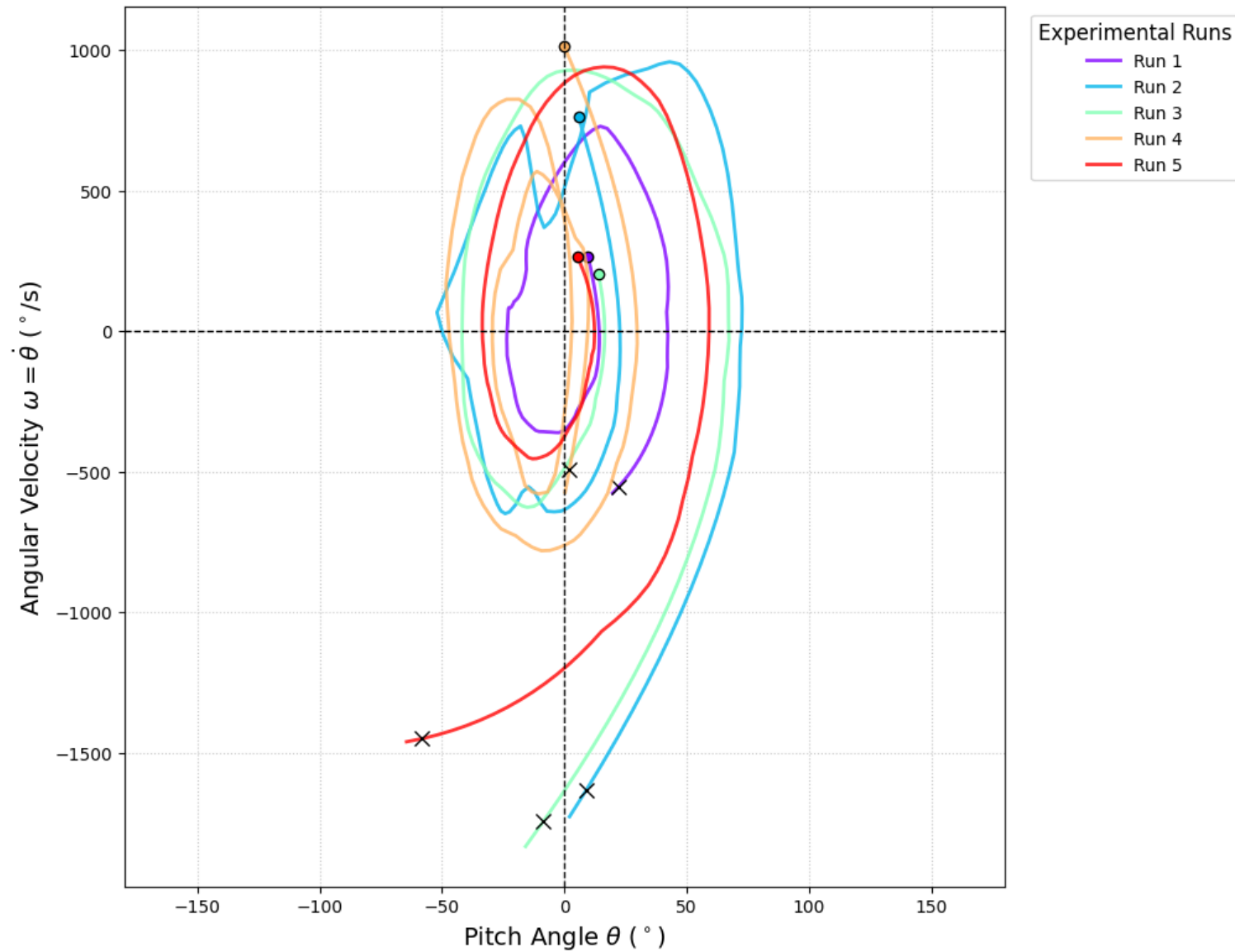
Coin Orientation

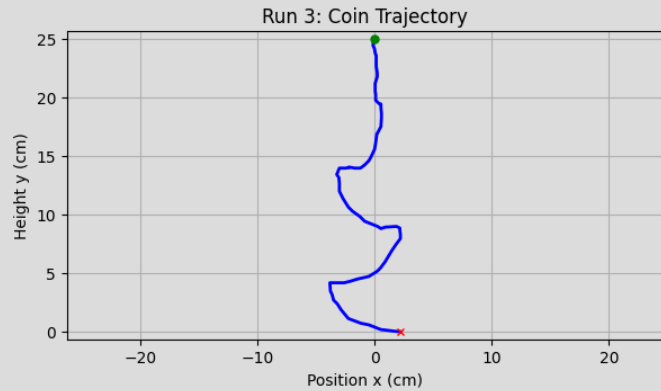
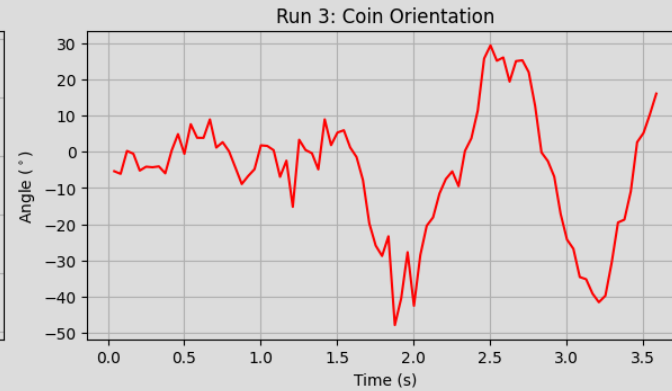


Chaotic

Quasi-periodicity

Phase Space Portrait: Angle vs. Angular Velocity



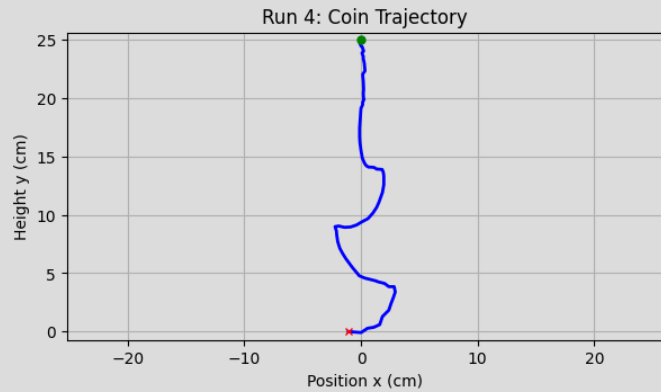
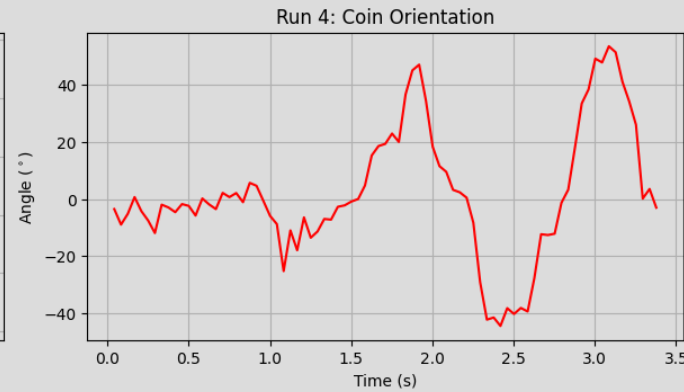


$$d = 3,8 \text{ cm}$$

$$t = 3 \text{ mm}$$

$$m = 4,1 \text{ g}$$

$$U = 6,4 \text{ cm/s}$$



- Coin swings back and forth
- Slow descend

